CAMBRIDGE
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SAT®
(Form Code A)

Cambridge Navigator Plus:
The Complete Explanation Guide
to the Retired Test

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5 Ways to Increase Score Gains Using Cambridge's *Navigator Plus*

Navigator Plus is Cambridge's complete explanation guide to a previously administered test. It includes explanations for each item on the test, categorization for each item, an answer key, and more.

The following list provides suggestions for implementing the Navigator into your program to increase score gains.

1. **Simulate test day as much as possible when proctoring tests.** Students will benefit from a testing experience that closely simulates what they will experience on test day. They will feel more confident if they know what to expect.

2. **Follow up when you receive your data.** Use the reports you receive from Cambridge to cover the items your class struggled as a group to answer (see the Error Analysis report). Taking this step within two weeks of administering the test will ensure that your students haven't forgotten the items you cover and will be able to learn from their testing experiences.

3. **Use the Pre-Assessment Item references in the *Victory* lesson to illustrate key points.** Your teacher's guide includes references to items on your pre-assessment that you can use as additional examples. Keep a copy of your pre-assessment test booklet handy so that you can cover these items with your students. Using pre-assessment items as additional examples helps students connect the concepts you are teaching with their test-day experiences.

4. **Don't forget to review the wrong answers.** Many explanations in this Navigator packet include references to each wrong answer choice. Students will benefit from reviewing why each wrong answer is wrong so that they can recognize what makes the right answer correct and use the process of elimination to eliminate similar wrong answers in the future.

5. **Pay attention to item categories.** Each item in this Navigator packet includes a category path that corresponds to the course concept outline in your *Victory* text as well as the categories listed in the Item Index of your *Victory* text. Use the Item Index to identify items students can use for further practice.

Category Paths

Throughout these explanations, each item includes a **Cambridge Category Path** which links the item to the Course Concept Outline in Cambridge's *Victory* series. For example:

> ### *Math: Multiple-Choice/Geometry/Triangles/Pythagorean Theorem*

An item with this particular category path is found in the Math Test (these items have a Level 1 label of "Math: Multiple-Choice" or "Math: Student-Produced Response") and tests students' knowledge of geometry (Level 2 of the category path), more specifically of triangles (Level 3), and even more specifically of the Pythagorean theorem (Level 4). The *Victory* Math Lessons include a section on the Pythagorean theorem, which you can find by referencing the Course Concept Outline at the beginning of the mathematics section in the *Victory* book. Additionally, you can find items testing geometry, triangles, or the Pythagorean theorem using the Item Index at the end of the *Victory* Student Text and Teacher's Guide.

Answer Key

DIRECTIONS: For items answered <u>correctly</u>, circle the answer, then check any corresponding shaded box(es). Total the number of circled answers to determine the raw score for the test section. Total the number of checkmarks for each of the subscores and cross-test scores to determine each raw subscore and raw cross-test score.

Section 1: Reading

		Subscores		Cross-Test Scores	
		WC	CE	S	H/S
1.	B			▓	
2.	C			▓	
3.	D		▓	▓	
4.	D			▓	
5.	A	▓		▓	
6.	B			▓	
7.	D		▓	▓	
8.	A			▓	
9.	A	▓		▓	
10.	D			▓	
11.	B			▓	
12.	A				▓
13.	A		▓		▓
14.	C	▓			▓
15.	C				▓
16.	D		▓		▓
17.	A				▓
18.	B	▓			▓

		Subscores		Cross-Test Scores	
		WC	CE	S	H/S
19.	A				▓
20.	D				▓
21.	C				▓
22.	B				
23.	D		▓		
24.	B				
25.	D		▓		
26.	B				
27.	C	▓			
28.	C	▓			
29.	B				
30.	B				
31.	A				
32.	D			▓	
33.	D			▓	
34.	C		▓	▓	
35.	A	▓		▓	
36.	B			▓	

		Subscores		Cross-Test Scores	
		WC	CE	S	H/S
37.	B		▓	▓	
38.	C	▓		▓	
39.	A			▓	
40.	D			▓	
41.	D			▓	
42.	B				▓
43.	D				▓
44.	B	▓			▓
45.	C				▓
46.	B				▓
47.	B		▓		▓
48.	C	▓			▓
49.	D				▓
50.	B				▓
51.	D		▓		▓
52.	A				▓

Raw Score: _____ /52

Section 2: Writing and Language

		Subscores		Cross-Test Scores	
		WC	CE	S	H/S
1.	A				
2.	D				
3.	A				
4.	A		▨		
5.	B				
6.	D		▨		
7.	D	▨			
8.	D				
9.	C				
10.	C	▨			
11.	A				
12.	C	▨			
13.	A		▨		
14.	C				
15.	B				

		Subscores		Cross-Test Scores	
		WC	CE	S	H/S
16.	B				
17.	D				
18.	B				
19.	B				
20.	C				
21.	C	▨			
22.	D		▨		
23.	A	▨		▨	
24.	A			▨	
25.	B				
26.	B				
27.	A				
28.	B			▨	
29.	C				
30.	D		▨	▨	

		Subscores		Cross-Test Scores	
		WC	CE	S	H/S
31.	C				
32.	D	▨		▨	
33.	C		▨	▨	
34.	B				
35.	B				
36.	D				
37.	D	▨			▨
38.	C				
39.	C				▨
40.	C				
41.	C				▨
42.	C	▨			▨
43.	D		▨		▨
44.	A		▨		▨

Raw Score: _____/44

Evidence-Based Reading and Writing Subscores

Words in Context (WC): _____/18   Command of Evidence (CE): _____/18

Section 3: Math—No Calculator

		Cross-Test Scores	
		S	H/S
1.	B		
2.	B		
3.	A		
4.	C		
5.	B		
6.	C		
7.	C		

		Cross-Test Scores	
		S	H/S
8.	A		
9.	B		
10.	C		
11.	D		
12.	C	▨	
13.	A		
14.	B		

		Cross-Test Scores	
		S	H/S
15.	D		
16.	72		
17.	3		
18.	24		
19.	3		
20.	78		

Section 4: Math—Calculator

#	Ans.	Cross-Test Scores S	Cross-Test Scores H/S	#	Ans.	Cross-Test Scores S	Cross-Test Scores H/S	#	Ans.	Cross-Test Scores S	Cross-Test Scores H/S
1.	A			14.	A	▓		27.	C	▓	
2.	D			15.	C	▓		28.	C		
3.	D		▓	16.	D	▓		29.	C		
4.	C			17.	D			30.	D		
5.	D			18.	A		▓	31.	27		▓
6.	B	▓		19.	C		▓	32.	5		
7.	D			20.	D		▓	33.	2/3, 4/6, 0.66		
8.	D			21.	A			34.	20		
9.	B		▓	22.	B			35.	7		
10.	C		▓	23.	D			36.	75		
11.	C		▓	24.	B			37.	1		
12.	B			25.	C	▓		38.	4		
13.	A			26.	B	▓					

Math Raw Score (total of calculator and no-calculator sections): _____ /58

Cross-Test Scores (All four test sections)

Science (S): _____ /35   History/Social Studies (H/S): _____ /35

Explanations

Section 1: Reading

Questions #1–11

1. (B) *Reading/Natural Sciences/Main Idea.* The main purpose of the passage is to review the findings of some research on animal behavior and suggest that this may have implications for the study of depression in humans. (B) neatly restates this. (A) can be overruled since the author proposes no cure and even notes that there are complex issues remaining to be solved. (C) is incorrect since the author does not criticize any experiments. It is important to recognize that in the fourth paragraph, the author is not being critical of any study in which rats were immersed in cold water but rather is anticipating a possible interpretation of those results and moving to block it. So, the author's criticism is of a possible interpretation of the experiment, not of the experiment itself or of the results. In any event, that can in no way be interpreted as the main theme of the passage. (D) is way off the mark. Though one might object to the use of animals for experimentation, that is not a burden the author has elected to carry.

2. (C) *Reading/Natural Sciences/Implied Idea.* In the final paragraph, the writer notes that inescapable noise or unsolvable problems produce human behavior similar to that induced in the lab subjects but adds that humans are much more complex.

3. (D) *Reading/Natural Sciences/Textual Evidence.* As noted in the explanation to the previous question, the explanation is found in the final paragraph, and (D) contains the relevant text excerpt.

4. (D) *Reading/Natural Sciences/Development.* The author introduces the questions in the fourth paragraph to anticipate a possible objection: perhaps the animal's inability to act was caused by the trauma of the shock rather than the fact that it could not escape the shock. The author then lists some experiments whose conclusions refute this alternative explanation. (A) is incorrect since the question represents an interruption of the flow of argument, not a continuation of the first paragraph. (B) represents a misreading of the passage. (C) can be eliminated since the author seems to think that the various questions asked do have an answer.

5. (A) *Reading/Natural Sciences/Vocabulary.* The author contrasts "nonaversive stimulus" with "uncontrollable aversive events" used in the experiments, namely, electric shocks. So the critical difference must be the pain—it is present in the shock experiments and not in the nonaversive parallels. This is further supported by the example of a nonaversive parallel, the uncontrollable delivery of food. So the relevant difference is articulated by (A). (B) is incorrect since none of the stimuli, even though they may not permanently injure, are enjoyable for the laboratory animals. (C) is incorrect because the events are all significant. (D) is incorrect since the author contrasts the nonaversive stimuli with the traumatic stimuli.

6. (B) *Reading/Natural Sciences/Explicit Detail.* According to the explanation of the experimental neurosis paradigm set out in paragraph six, as animals are presented with increasingly similar choices, they exhibit abnormal behavior such as agitation and then lethargy.

7. (D) *Reading/Natural Sciences/Textual Evidence.* The experimental neurosis paradigm is explained in paragraph six and is the basis for the correct answer to the preceding question.

8. (A) *Reading/Natural Sciences/Development.* The author raises a question in paragraph four in order to anticipate a possible objection; namely, that the shock, not the unavoidability of it, caused inaction. The author then offers a refutation of this position by arguing that we get the same results using similar experiments with nonaversive stimuli. Moreover, if trauma of shock caused the inaction, then we would expect to find learned helplessness induced in rats by the shock, regardless of prior experience with shock. The "mastery effect," however, contradicts this expectation. This is essentially the explanation provided in (A). (B) is incorrect since the author does not mention this until the end of the passage. (C) can be eliminated since the "mastery effect" reference is not included to support the conclusion that neurochemical changes cause the learned helplessness. (D) is incorrect, for though the author makes such an assertion, the "mastery effect" data is not adduced to support that particular assertion.

9. (A) *Reading/Natural Sciences/Vocabulary.* In the relevant paragraph, the author explains that prior experience can be used to immunize against learned helpless. The "immunization" manifests itself as a "mastery effect," that is, the rats work longer and harder to avoid the shock.

10. (D) *Reading/Natural Sciences/Application.* The author closes with a disclaimer that the human cognitive makeup is more complex than that of laboratory animals, and that for this reason, the findings regarding learned helplessness and induced neurosis may or may not be applicable to humans. The author does not, however, explain what the differences are between the experimental subjects and humans. A logical continuation would be to supply the reader with this elaboration. By comparison, the other answer choices are less likely. (B) is unlikely since the author begins and ends with references to human depression, and that is evidently the motivation for writing the article. (C) is not supported by the text

since it is nowhere indicated that any such experiments have been undertaken. Finally, (A) is perhaps the second-best answer; its value is that it suggests the mechanism should be studied further. But the most important question is not how the mechanism works in rats but whether that mechanism also works in humans.

11. **(B)** *Reading/Natural Sciences/Implied Idea.* The experimenters were interested in whether or not the test subjects would make the effort to escape the shocks or simply endure them. Adjusting the height of the barrier would make escape more or less difficult, helping to determine whether those that escaped did so at random or because they had learned to find an avoidance mechanism.

Questions #12–21

12. **(A)** *Reading/Social Studies/Implied Idea.* In paragraph one, the author states that cartoon violence has three cues, make-believe violence two cues, and an acted violence only one cue. Presumably, the easiest to distinguish is the one with the most cues.

13. **(A)** *Reading/Social Studies/Textual Evidence.* As noted in the explanation to the previous question, presumably the form with the greatest cues is the most readily identifiable. And (A) here provides the explicit statement of that presupposition.

14. **(C)** *Reading/Social Studies/Vocabulary.* This item tests a common word, so look for the meaning appropriate to the context. The author means to say that an aggressive action might happen only when it would otherwise be expected or normal.

15. **(C)** *Reading/Social Studies/Implied Idea.* In the fifth paragraph, the passage states that depiction of violence might have a short-term effect on a child, perhaps for 15 or 20 minutes after a child has viewed the violence. If, in that time frame, the child is then put in a situation that calls for aggressive behavior, he or she might act out aggressively. However, the author also points out that this effect tends not to last longer than an hour. Therefore, the author would conclude the child's behavior after a couple of hours is not likely to be affected by the earlier viewing of violence.

16. **(D)** *Reading/Social Studies/Textual Evidence.* As the excerpt makes clear, the author believes that any instigational effect would be short-term, less than 20 minutes.

17. **(A)** *Reading/Social Studies/Implied Idea.* The author theorizes that the behavior of children may be changed by watching violence on television, but not for the reason that most people think. According to the author, it is not the violence itself that gets kids worked up but the energy level in the depiction. The author goes on to explain that anything that gets a kid excited may have the same effect, even agitating the child by turning off the TV set.

18. **(B)** *Reading/Social Studies/Voice.* In this case, the author is clearly an opponent of the view that fantasy violence on television adversely affects children. (Though the author does say that violence may inspire teenagers or adults to commit violent acts.) In the last paragraph, the author takes on those who hold to the commonly accepted view and accuses them of using television as a scapegoat when the real causes of violence in our society are cultural. The use of the word "scapegoat" is strong and indicates that the author objects to that view.

19. (A) *Reading/Social Studies/Main Idea.* First, look at the first word of each answer choice and then look at the rest of each choice. The best first word to describe the purpose of the passage is probably "correct" since the author challenges a commonly accepted position. Looking at the rest of (A), it is clear that (A) is the correct answer: the author rejects the common notion that television violence affects the behavior of children.

20. (D) *Reading/Social Studies/Data Presentations.* The graph indicates that both parents and children agree that the child feels more violent immediately after viewing TV violence. This provides some support to statement (D): violent portrayals could be dangerous in the short run. Granted the data do not prove the point, but at least the data seem relevant to this claim. The data do not seem to support the other statements at all.

21. (C) *Reading/Social Studies/Application.* According to paragraph one, the distinguishing features of make-believe are humor and a fictional setting, those described by (C).

Questions #22–31

22. (B) *Reading/Literary Fiction/Implied Idea.* The first half of the passage tells us that Duane is fleeing from something but provides no details. Paragraph three, however, makes it clear that the event had taken place earlier in the day, so it was a single incident that had wrought this change in Duane, not a lifetime of crime. Duane's reflection in paragraph three presents him as an ordinary person whose life has suddenly changed and who feels that he must now live like an outlaw.

23. (D) *Reading/Literary Fiction/Textual Evidence.* (D) summarizes the dilemma that Duane finds himself in. He is not basically a bad person—he loathes bad people—but he will be forced to live either alone or among men he finds obnoxious. Or, if he chooses an honest living, he must be careful to protect his identity.

24. (B) *Reading/Literary Fiction/Implied Idea.* Everything in the first paragraph makes it clear that Duane is looking for a safe place to camp. He avoids the sites he is familiar with; he moves far off the trail; he finds a secluded valley that is undercover.

25. (D) *Reading/Literary Fiction/Textual Evidence.* The text states that Duane passed by campsites that were already familiar to him, looking for something more secluded, and chose a site far from the trail under cover.

26. (B) *Reading/Literary Fiction/Implied Idea.* The first paragraph says that Duane rode his horse a considerable distance and made a change of direction to avoid encountering other people. Everything in this part of the selection conveys a sense of urgency to put distance between himself and the place where an event occurred. After Duane sets up a camp, he then begins to think on the significant event of that day and its implications for his life.

27. (C) *Reading/Literary Fiction/Vocabulary.* At the end of the first paragraph, Duane makes a substitute hobble using a length of rope. It hampers the horse's movement but doesn't confine it to one spot, and it only requires a few feet of rope, so (C) must be correct.

28. **(C)** *Reading/Literary Fiction/Vocabulary.* "Vigilant" means "alert," "cautious," or "wary." In this context, Duane is obliged to be vigilant always against the possibility that someone will find him and hold him responsible for whatever he did earlier in the day.

29. **(B)** *Reading/Literary Fiction/Explicit Detail.* All four of the choices are types of terrain mentioned in the passage, but the first paragraph states that Duane chose the low hills that he had earlier seen from the distance.

30. **(B)** *Reading/Literary Fiction/Implied Idea.* The fact that he was nervous surprised Duane because he was already starting to develop new habits, the habits of someone perhaps pursued and concealing his identity, but not yet aware of that fact.

31. **(A)** *Reading/Literary Fiction/Implied Idea.* Duane selected the campsite for its solitude. He is hiding there from any pursuers. As darkness falls and the birds and insects become quiet, the place seems even more remote than during the day. This gives Duane a feeling of even greater safety.

Questions #32–41

32. **(D)** *Reading/Natural Sciences/Main Idea.* The first and last sentences of the selection effectively provide the answer. (A) is too narrowly drafted because the author does not only discuss the containment issue (the second issue). (B) goes beyond the explicit scope of the selection because competitiveness is not mentioned. (C) likewise is wide of the mark. The author focuses on physical forces, not energy efficiency.

33. **(D)** *Reading/Natural Sciences/Implied Idea.* In the third paragraph, the author explains that the energy stored by a flywheel is a function of its mass and speed, and that a lightweight flywheel would have to spin faster in order to store the same energy as a heavier one. So, you can infer that if the masses are different but the stored energy is the same, the lighter wheel is spinning faster—and that's (D). (A) is a confused reading of the first paragraph, and that information doesn't help answer the question that is asked. Similarly, (B) and (C) come out of the second paragraph and aren't relevant here.

34. **(C)** *Reading/Natural Sciences/Textual Evidence.* (B) provides explicit textual support for the explanation given to #33: lightweight rotating at higher speed = heavyweight rotating at lower speed.

35. **(A)** *Reading/Natural Sciences/Vocabulary.* If you know in advance that the meaning of "benign" is "nonthreatening" or "not dangerous," (A) is the obvious choice. The correct answer can also be found within the context of the third paragraph. The author is discussing what happens when one of these rapidly spinning wheels breaks up. If it's made of heavy metal, the result is a lot of shrapnel flying everywhere. But if it's made of graphite fiber, then the result is a tangled bunch of threads—much less dangerous than the alternative.

36. **(B)** *Reading/Natural Sciences/Explicit Detail.* The answer to this item is contained in the second paragraph: the net effect of the two modules on the car is nearly zero when they are spinning in opposite directions, (B).

37. **(B)** *Reading/Natural Sciences/Textual Evidence.* As noted in the explanation to the previous question, the answer is contained in the second paragraph.

38. **(C)** *Reading/Natural Sciences/Vocabulary.* The second sentence of paragraph two refers to gyroscopic forces that come into play when the vehicle is turning. The author discusses this issue and concludes in the final sentence of the paragraph that the net effect would be zero if the pairs are spun in opposite directions. So the net effect discussed is the effect when the car is turning.

39. **(A)** *Reading/Natural Sciences/Development.* The author introduces the idea of EMBs for use in cars and describes generally how they would work. Then, in the second paragraph, the author states that they pose "two special problems." The author describes those problems and explains how they are minimized by using different technological solutions, (A). The other choices simply do not fit the logical development of the passage.

40. **(D)** *Reading/Natural Sciences/Data Presentations.* data in the graph show that that alloy steel flywheels are good for only 50 Wh/kg or so, while graphite fiber flywheels may generate up to 500+ Wh/kg, or nearly ten times more Wh/kg than the alloy steel flywheels.

41. **(D)** *Reading/Natural Sciences/Main Idea.* In the first paragraph, the author is primarily explaining how the EMB works in an electric car. To the extent that the ideas mentioned by the other choices are found in the passage, they are discussed in other paragraphs.

Questions #42–52

42. **(B)** *Reading/Social Studies/Explicit Detail.* In the last paragraph, and the last sentence in particular, the author of the first passage states that the interdisciplinary approach used by Turner was a new technique. (B) best captures this idea. As for (A), the same paragraph states specifically that the reliance on political history was characteristic of history prior to Turner. As for (C), although Turner made the original presentation at a conference, the passage does not say that presenting was a technique of study. While Turner used the opportunity to present his new theory, he could equally well have published an article or made an informal presentation to colleagues. As for (D), the first passage doesn't enter into such a debate, though you will find some mention of this in the second passage. But because the information appears in the second passage, it cannot be an answer to this Explicit Detail item about the first passage.

43. **(D)** *Reading/Social Studies/Voice.* The author of Passage 1 evidently approves of Turner's work. The passage says that it had great influence, that it was original, and that it used a novel approach. That's a pretty good review. As for (A), while the author allows that Turner's thesis was not immune to debate or even criticism, this does not mean that the author was "skeptical" of the work itself. After all, it could turn out to be that Turner's conclusions are ultimately false; but the groundbreaking approach and radical theory would still have value. As for (B), the thesis is not treated negatively, so "condescending" cannot be used to express the author's attitude. Finally, as for (C), the author takes a pretty strong position, so "noncommital" is not a good description.

44. **(B)** *Reading/Social Studies/Vocabulary.* Because "grand" is a word with some common meanings, you can pretty much discount any choices that use these more common synonyms. That would certainly eliminate (C). Instead, the author is using the word "grand" in a derivative sense to mean large or great or overall. Turner's thesis did try to be comprehensive, accounting for the uniquely American character. As for (A), though the author allows that Turner's thesis was not perfect, line 13 is not where that discussion occurs. And (D) must be wrong since Turner's thesis was not tentative.

45. **(C)** *Reading/Social Studies/Development.* The author of Passage 2 discusses the limitations of Turner's theory, and one of the most important of these is its attempt to explain everything American in terms of

the frontier. At the referenced lines, the author lists some other very important historical factors in order to show that the frontier could not have been the entire story. As for (A), this is the topic introduced at the end of that paragraph and developed in the following paragraphs, but it is not an answer to this question. As for (B), even granting that this statement is correct, it is not an answer to this question. For example, the author mentions the Civil War in order to show that Turner's thesis was too limited, not that traditional histories were too limited. As for (D), this is a point that is raised in the fourth paragraph, so it is not an answer to the question asked about the second paragraph.

46. (**B**) *Reading/Social Studies/Voice.* The author of Passage 2 is critical of the frontier thesis, but you'll notice that the criticisms all deal with Turner's ideas. For example, Turner thought that the frontier offered free land, but the author of Passage 2 argues that he was wrong because the land was already used by indigenous people. So, while the passage criticizes Turner's idea, it doesn't criticize Turner himself. Thus, (C) and (D) are wrong, and (B) is correct. As for (A), the author says that the thesis has "rightfully" been abandoned because of its weaknesses.

47. (**B**) *Reading/Social Studies/Textual Evidence.* Perhaps the best summary statement of the second author's thesis is that the frontier thesis has been "rightfully abandoned." The author goes on to provide a refutation in detail, but as evidence of the writer's general attitude toward Turner's thesis, "rightfully abandoned" is a pretty good summary.

48. (**C**) *Reading/Social Studies/Vocabulary.* You get the information you need to answer the question from the discussion about the significance of the Indian Wars. Turner claimed that the land was free, but in reality, it was necessary to pursue a policy of military aggression to secure the land. So, when the author writes that the wars "belie" the free land theory, the author means "prove false." (A) is a distracting choice, but don't be misled by the superficial connection between "lie" and "untruth." In this context, the phrase is "prove to be false," not "lie about." As for (B) and (D), while these are phrases that relate generally to the idea of debating the merits of a theory, they don't focus on the connection between the wars and the free land thesis.

49. (**D**) *Reading/Social Studies/Explicit Detail.* The key word other than "EXCEPT" in this item is "both"; therefore, the correct answer choice is the only one that is *not* mentioned in *both* passages. Both authors mention (A), (B), and (C). However, "nationalism" is only mentioned in the first passage.

50. (**B**) *Reading/Social Studies/Implied Idea.* In the final paragraph, the author argues that group activity—migration of extended family groups, corporate undertakings such as the railroads, government military intervention—and not individual action settled the frontier.

51. (**D**) *Reading/Social Studies/Textual Evidence.* The final paragraph of Passage 2 contains a fairly thorough refutation of Turner's thesis that rugged individuals "tamed the frontier." And the first sentence of that paragraph provides a pretty good summary of its development.

52. (**A**) *Reading/Social Studies/Application.* To a certain extent, any weakening of Turner's theory would have implications for all aspects of the theory. So, you might argue that the referenced evidence, in some way, tends to show that people from different regions did not mix at the frontier because the frontier was not quite as well-defined as Turner thought. But that's a pretty feeble point, so (D) is wrong. You can apply similar reasoning to (B) and (C). The best answer here is (A). The "safety valve" point, as explained in Passage 1, maintained that people who were dissatisfied with life in the urban areas could simply pack up and move to the country because there was land for the claiming. If the "free land" thesis is false, then the "safety valve" thesis must also be false.

Section 2: Writing and Language

Passage 1

1. **(A)** *Writing and Language/Standard English Conventions/No Change.* The original sentence is correct. The subject of "are" is "symbols and meanings," and a plural subject requires a plural verb. On first reading, it might be unclear whether the use of the present tense is correct, but after reading the next few sentences, it is clear the author intends to write in the present tense, so the original sentence is correct.

2. **(D)** *Writing and Language/Standard English Conventions/Sentence Structure/Problems of Coordination and Subordination*. The underlined portion contains a transitional word that does not correctly express the relationship between the elements of the sentence. "Meanwhile" is used to imply events that occur simultaneously. For example, "Meanwhile, back at the ranch house, the cook was making dinner, unaware that the bandits were ready to strike." "While" is used to create a contrast; (D) creates this necessary contrast. As for (B) and (C), "because" is incorrect; the author does not intend to imply a causal connection between the two ideas.

3. **(A)** *Writing and Language/Expression of Ideas/Organization/Paragraph-Level Structure*. The defining characteristic of the first paragraph is the series of parallel characteristics: circular halo vs. halo with cross; bare feet vs. Virgin Mary; Saint Peter vs. Saint Paul; etc. The addition completes the pair: earth vs. heaven.

4. **(A)** *Writing and Language/Expression of Ideas/No Change*. The main idea of the first paragraph is that art of the Middle Ages is a sacred script. The sentence that begins with the underlined portion should be part of the first paragraph because it is an example of the symbols and meanings that characterize this art as a sacred script. In other words, the example provides proof for the position the author develops in the first paragraph. As for the other choices, (C) introduces a problem of logical expression—several "towers" would not have a single window, and in (D), the expression "having a tower" seems to be an adjective phrase, but there is nothing for it to modify.

5. **(B)** *Writing and Language/Standard English Conventions/Grammar and Usage/Nouns and Noun Clauses.* The logic of the original sentence is distorted. Specifically, something is missing, and (B) supplies the missing element: the relative pronoun "who" to function as the subject of the verb "had revived." (C) uses the wrong relative pronoun ("who" is preferred for referring to people) and introduces an incorrect apostrophe. (D) also incorrectly uses the possessive.

6. **(D)** *Writing and Language/Expression of Ideas/Strategy/Appropriate Supporting Material*. The second paragraph discusses numbers in the art of the Middle Ages. The author gives several examples to prove the point that mathematics was an important element. The underlined sentence, however, talks about music. As used, the sentence doesn't support the author's point, isn't parallel to the other example, and doesn't belong in the paragraph.

7. **(D)** *Writing and Language/Expression of Ideas/Style/Conciseness*. The original underlined version is needlessly wordy. "Can be seen" already includes the notion of "viewers," so the phrase can be deleted without losing any meaning.

8. (D) *Writing and Language/Standard English Conventions/Grammar and Usage/Pronoun Usage.* The original contains two errors. First, the relative clause introduced by "who" cannot serve as the object of the verb "symbolizes" because "who" introduces an adjective and we need some sort of noun. So a good alternative is to use the demonstrative pronoun "those" to function as the object, which is then modified by the relative clause introduced by "who." Second, the verb tenses are not parallel: . . . are lost and . . . have been saved. The first is present, the second imperfect. (D) corrects both errors.

9. (C) *Writing and Language/Standard English Conventions/Punctuation/Commas.* Remember that commas are needed to show interruptions in the flow of a sentence, interruptions such as introductory phrases and asides:

Introductory phrase: Hoping to impress the job interviewer, Shareen wore her brand-new Gucci suit.

Aside: Xavier, like his sister Maya, looks very much like their father.

In the item, the aside "like nature" must be set off by a pair of commas, one showing the start of the aside and the other showing its end.

10. (C) *Writing and Language/Expression of Ideas/Style/Precision.* The passage means to say that the works of artists who followed the rules were almost automatically good or even better, despite the skill of the artists, and the best choice to express that thought is "elevated."

11. (A) *Writing and Language/Expression of Ideas/Strategy/Effective Concluding Sentence.* Adding the suggested sentence to the passage serves two useful purposes. First, at that point, the writer distinguishes the Renaissance artists who break with tradition as either great or not so great. The great ones, according to the writer, produce paintings that are equal to those of the masters of the Middle Ages, but the works, for all the risk, aren't really better. Adding the sentence completes the comparison by discussing the Renaissance artists who break with tradition but aren't really all that talented: their works are just mediocre. That is sufficient reason to add the sentence. Now, once the sentence is added, the two-part comparison really wraps up the passage: the conventions were so powerful that later artists, whether great or ordinary, who broke with tradition did not at first make a lot of progress.

Passage 2

12. (C) *Writing and Language/Expression of Ideas/Style/Tone.* The original, (B), and (D) use expressions that are informal usage. The more neutral and detached "express concern" is more consistent with the academic tone of the rest of the passage.

13. (A) *Writing and Language/Expression of Ideas/Strategy/Appropriate Supporting Material.* This item requires you to show that you understand the writer's argument and its structure. The writer distinguishes three senses of "shortage," and in the next paragraph talks about shortages that can be corrected by rising prices. This is the second type of shortage listed.

14. (C) *Writing and Language/Standard English Conventions/Punctuation/Commas.* This item tests the use of commas, and the underlined portion includes two different usages. First, the series "academic, industrial, and government employees" is correctly punctuated. Remember that each element of a series with three or more elements is followed by a comma to set it off from the following element—except the last, of course. The serial comma or Oxford comma is optional; "academic, industrial and government employees" is also acceptable. So the original is correct so far as this practice is concerned. But it is also

required that the end of an introductory dependent clause be marked with a comma, and the original is incorrect on this score. You need a comma following "employers," not because "employers" is an element of the series but because "employers" is the last word in the dependent clause.

15. **(B)** *Writing and Language/Standard English Conventions/Sentence Structure/Faulty Parallelism.* The two verbs here must both be conjugated forms and in the present tense. This is required by the sense of the sentence (they "observe," and they "consider") and not some requirement of parallelism. Thus, (C) is wrong because it imposes an unneeded parallelism that distorts the logic of the sentence.

16. **(B)** *Writing and Language/Expression of Ideas/Strategy/Appropriate Supporting Material.* This item asks for the correct conclusion to be drawn from the paragraph. The writer argues that a shortage will drive prices up. In the case of a particular kind of worker, a shortage will cause wages to rise as employers try to fill their empty positions. Then, when those in the education pipeline see that wages are rising, some will choose that area for a career, and more workers will be available, reducing the shortage. As the shortage is eliminated, prices will no longer rise.

17. **(D)** *Writing and Language/Standard English Conventions/Grammar and Usage/Subject-Verb Agreement.* The sentence needs a conjugated (main) verb that agrees with the singular subject "aid." The underlined portion has two verbs, and they should be "lowers" and "attracts."

18. **(B)** *Writing and Language/Expression of Ideas/Organization/Paragraph-Level Structure.* The underlined sentence poses a question that is answered by the rest of the paragraph. Therefore, it should be the first sentence.

19. **(B)** *Writing and Language/Expression of Ideas/Strategy/Data Presentation.* The graph shows that professions such as doctors and lawyers earn more than their counterparts in science and engineering. This suggests that there are fewer professionals for the jobs available than there are scientists and engineers for those jobs.

20. **(C)** *Writing and Language/Standard English Conventions/Sentence Structure/Comma Splices.* As written, the sentence seems to be a comma splice. Ordinarily, you might correct a comma splice by adding a conjunction:

> . . . graduates, and most work . . .

But that raises a problem of coordination because the two resulting clauses are not making statements of equal significance. So, perhaps the sentence intends a relative clause:

> . . . graduates, most of whom work . . .

That would solve the problem of the original while properly expressing the dependence of the relative clause, but it is not a possible choice. And (B) isn't equivalent. The remaining possibility seems to be that the writer intends an adjective phrase:

> . . . graduates, mostly working . . .

And that is answer (C).

21. (C) *Writing and Language/Standard English Conventions/Grammar and Usage/Pronoun Usage.* What is intended by the writer is a possessive pronoun that refers to "expatriates." The correct form is "their."

22. (D) *Writing and Language/Expression of Ideas/Strategy/Main Idea.* This is a main idea question. The writer has argued that there is no real shortage of scientists and engineers using data that shows that such workers are paid less than their professional counterparts. Lower compensation means that firms are able to find all the workers they need and so don't have to pay higher compensation. If the temporary workers program is expanded, then more foreign workers will apply for the jobs and are willing to work for less since "less" here is "more" than they would earn at home.

Passage 3

23. (A) *Writing and Language/Expression of Ideas/No Change* "Pseudonym" or assumed name is what the writer intends here.

24. (A) *Writing and Language/Expression of Ideas/No Change.* "However" is a good conjunctive adverb for showing a contrast between two ideas. Here, the contrast is between the early days of bitcoin and its more recent history.

25. (B) *Writing and Language/Standard English Conventions/Sentence Structure/Comma Splices.* The original sentence contains a comma splice—two independent clauses jammed together using just a comma. There are at least three ways of correcting this error:

view; peer-to-peer

view. Peer-to-peer

view, and peer-to-peer

(B) joins the two clauses using a semicolon to mark the end of one clause and the beginning of the other. (C), while grammatically correct, would create a contrast that would change the meaning of the sentence.

26. (B) *Writing and Language/Standard English Conventions/Grammar and Usage/Faulty or Illogical Comparisons.* The problem with the original is an illogical comparison. The original attempts to compare the controlling agencies of earlier currencies with the bitcoin network: agency vs. network. But as the passage makes clear, those are two distinct ideas.

27. (A) *Writing and Language/Standard English Conventions/No Change.* This is a fairly standard item involving punctuation. Asides such as definitions and appositives need to be separated from the main body of the sentence using commas, one to mark the beginning and one to mark the end of the phrase.

28. (B) *Writing and Language/Expression of Ideas/Strategy/Effective Transitional Sentence.* This can be a fairly tricky item. The author announces in the fourth paragraph that there are three ways to obtain bitcoins. The first is explained in that same paragraph and is flagged "first" for the reader. The second is also flagged ("second"), and a new paragraph signals the start of the discussion of the second. Where is the third? Mining is the third, and it should be signaled for the reader.

29. **(C)** *Writing and Language/Standard English Conventions/Sentence Structure/Misplaced Modifiers.* The problem with the original is that the introductory modifier seems to apply to "math problem" when the writer intends for it to apply to "miners." (C) corrects this problem by repositioning "miners" to make it clear that it is the miners who must solve the problem. Another way of looking at the sentence is to say that it awkwardly uses the passive voice.

30. **(D)** *Writing and Language/Expression of Ideas/Organization/Paragraph-Level Structure.* This item tests whether or not the reader understands the sequence of steps that make up the paragraph. The certification of the correctness of the work is the final step in the mining process. At that point the miner is awarded the 25 bitcoins for having done the work correctly.

31. **(C)** *Writing and Language/Standard English Conventions/Sentence Structure/Misplaced Modifiers.* The problem with the underlined part of the original is that the modifiers are not presented in a logical order. The author means to say that bitcoin remains a niche currency despite its growth. In the original, we actually have two independent modifiers: "since its inception" and "despite significant growth." (C) solves the problem by making "since its inception" modify "growth."

32. **(D)** *Writing and Language/Expression of Ideas/Style/Precision.* A niche is a shallow cavity in a wall that is often used to display a figurine or other ornament. Because such an object fits nicely in the cavity, *niche* is also used to refer to a comfortable resting place or particularly appropriate place for an object. A niche market is a small part of the market that is served by a company that provides goods or services that satisfy the peculiar demands of that small area.

33. **(C)** *Writing and Language/Expression of Ideas/Organization/Paragraph-Level Structure.* The sentence explains why bitcoins are subject to extreme swings in value. This general explanation should come before the author's specific examples of the bitcoin's volatility. So it should be placed fourth, before the sentence that begins "In mid-January 2015."

Passage 4

34. **(B)** *Writing and Language/Standard English Conventions/Grammar and Usage/Subject-Verb Agreement.* The underlined part of the original sentence contains an error of subject-verb agreement. The subject of the sentence is "leaders," a plural noun. So the verb must also be plural: were.

35. **(B)** *Writing and Language/Standard English Conventions/Grammar and Usage/Adjectives versus Adverbs.* The problem with the original is the use of the comparative *–er* form. The comparative form of adjectives is used to compare two ideas; the superlative or *–est* is needed for three or more. In this case, the passage announces that there were three foundational elements to the Meiji Restoration, so "oldest" is required.

36. **(D)** *Writing and Language/Standard English Conventions/Sentence Structure/Faulty Parallelism.* The problem with the original is a failure of parallelism: respect (noun) and worshipping (verb form). (D) corrects the problem by using parallel forms: respect for and worship of. Both are noun forms. (C) is close but fails because "respect" is forced to use the preposition "of," but "respect of" is not idiomatic.

37. (D) *Writing and Language/Expression of Ideas/Style/Precision.* This item pretty much reduces to a choice between (A) and (D). The weakness in (A) is that "perpetuated" is usually used to refer to a state of affairs that is undesirable, but there is no element of that here. Better would be "preserved."

38. (C) *Writing and Language/Standard English Conventions/Grammar and Usage/Nouns and Noun Clauses.* The problem with the original is that the participle form "having been" functions as an adjective. But the intention of the author is not to say that Neo-Confucianism would reinforce feudalism *because* it was once an official ideology. Rather, Neo-Confucianism had that effect because of the tenets of the religious-philosophical thought that it adopted. So what is required here is a relative clause: "Neo-Confucianism, which had been the official . . . ."

39. (C) *Writing and Language/Expression of Ideas/Strategy/Effective Transitional Sentence.* "Some" is either non-idiomatic or informal usage (depending on how you read it), so the resulting phrasing is not very precise. Substituting "another" makes the connection between the second tradition and the third tradition clear. The second tradition was a basic tenet of Confucianism, and the third tradition is yet another basic tenet of Confucianism.

40. (C) *Writing and Language/Standard English Conventions/Grammar and Usage/Subject-Verb Agreement.* The underlined portion of the sentence occurs in a clause with an inverted subject and verb. That is, the subject comes after the verb. The subject is "miles," so the plural "were" is needed.

41. (C) *Writing and Language/Expression of Ideas/Style/Idiomatic Expression.* The original is not only clumsy and informal usage ("got aided"), but there is a lack of parallelism of form between "got aided" and "encouraged." Both problems are eliminated by (C).

42. (C) *Writing and Language/Expression of Ideas/Style/Tone.* (A), (B), and (D) are all inconsistent with the formal tone of the passage. "Wholesale" expresses the idea of "widespread" or "unchecked."

43. (D) *Writing and Language/Expression of Ideas/Strategy/Main Idea.* This item asks what conclusion can be drawn from the information provided earlier in the paragraph. Earlier in the paragraph, the author states that Westernization eventually slowed, and the passage describes educational theory: initially influenced by the West but also stressing traditional values. The passage implies that art and literature were subject to similar forces: initially influenced by the West but traditional styles reasserted.

44. (A) *Writing and Language/Expression of Ideas/Strategy/Main Idea.* This question really is about the main point of the passage and asks you to say what sort of conclusion the evidence presented will support. The passage explains that the organizers of the Restoration were primarily interested in moving Japan into the twentieth century while still preserving stability in the country, so they mixed traditions with policies of modernization. The passage lists the various accomplishments such as building a transportation and communication infrastructure that moved Japan into the modern century. One paragraph states that tradition remained strong in a couple of areas. But the last sentence of the passage, as originally written, states unequivocally that Japan has moved into the twentieth century. (A) best expresses this balance.

Section 3: Math—No Calculator

1. **(B)** *Math: Multiple-Choice/Algebra/Manipulating Algebraic Expressions/Basic Algebraic Manipulations.* Note that $(b-a)=-(a-b)$ and $(c-b)=-(b-c)$. Thus, $(b-a)(c-b)=(-1)^2(a-b)(b-c)$, and so $(a-b)(b-c)-(b-a)(c-b)=0$.

   This item can also be solved by evaluating the given expression. Use the FOIL (First, Outer, Inner, Last) method for multiplying polynomials: $(a-b)(b-c)=ab-ac-b^2+bc$; $(b-a)(c-b)=bc-b^2-ac+ab$. Subtract the second expression from the first expression: $(ab-ac-b^2+bc)-(bc-b^2-ac+ab)=0$.

2. **(B)** *Math: Multiple-Choice/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions/Function Notation.* Substitute 3 for *x* in the given function and evaluate: $f(3)=3^2-3=6$. Repeat the procedure: $f(f(3))=f(6)=6^2-6=30$.

3. **(A)** *Math: Multiple-Choice/Algebra/Manipulating Algebraic Expressions/Factoring Expressions.* Factor the top and bottom expressions, and simplify: $\dfrac{m^2+mn}{m^2-n^2}=\dfrac{m(m+n)}{(m+n)(m-n)}=\dfrac{m}{m-n}$.

4. **(C)** *Math: Multiple-Choice/Algebra/Solving Algebraic Equations or Inequalities with One Variable/Equations Involving Absolute Value.* First, set up the derivative equations: $\dfrac{x-2}{3}=4$ or $\dfrac{x-2}{3}=-4$. And solve for *x*: $x-2=12\Rightarrow x=14$ or $x-2=-12\Rightarrow x=-10$. Then, check both solutions to eliminate any extraneous solutions: $\left|\dfrac{x-2}{3}\right|=4\Rightarrow\left|\dfrac{14-2}{3}\right|=4\Rightarrow|4|=4$ and $\left|\dfrac{x-2}{3}\right|=4\Rightarrow\left|\dfrac{-10-2}{3}\right|=4\Rightarrow|-4|=4$.

   Alternatively, this problem is a good opportunity to use the "test-the-test" strategy—it may be a faster solution than the algebra approach in this case.

5. **(B)** *Math: Multiple-Choice/Algebra/Solving Quadratic Equations and Relations.* The stem specifies that the solution set to the equation is $\{-2, 4\}$, so when *x* = –2 or *x* = 4, the equation is equal to zero: the factors are *x* + 2 and *x* – 4. Multiply the factors to determine *a*, *b*, and *c*: $(x+2)(x-4)=x^2-4x+2x-8=x^2-2x-8$, so *a* = 1, *b* = –2, and *c* = –8. Of the choices, only (B) is not true.

6. **(C)** *Math: Multiple-Choice/Coordinate Geometry/Slope of a Line.* The given functions $f(-1)=1$ and $f(2)=7$ define two coordinate points included in the graph of $f(x)$: $(-1,1)$ and $(2,7)$. Therefore, the slope of the line is $m=\dfrac{7-1}{2-(-1)}=\dfrac{6}{3}=2$.

7. **(C)** *Math: Multiple-Choice/Algebra/Manipulating Algebraic Expressions/Factoring Expressions.* As for (A), $p(x)=x^3+x^2-4x-4$, so it is a polynomial of the third degree and has three unique factors. As for (B), the zeroes of a polynomial are the values of x for which the expression equals zero, so test $x = 1$: $1(1+1-4)-4=-2-4=-6$, which isn't zero, so (B) is false. As for (C), plug –1 for x in $p(x)$ to determine if it equals 0: $(-1)[(-1)^2+(-1)-4]-4=(-1)(-4)-4=0$, so (C) is true. Note that (D) is false because substituting 0 for x in $p(x)$ returns a value of –4, not 0.

8. **(A)** *Math: Multiple-Choice/Coordinate Geometry/The Coordinate System.* A point where two lines on a graph cross is called a point of intersection. Two graphs cross when their y-values, at a single value of x, are equal. To find the points of intersection for two equations, simply set the two equations equal and solve for x. In this instance, $x=m$. So, $x^2-4=-x^2+4\Rightarrow 2x^2-8=0\Rightarrow x^2-4=0\Rightarrow x^2=4\Rightarrow x=\pm 2$. If $m>0$, then $m = 2$.

Another way to solve the problem is to set one of the equations equal to zero: $(m,0)\Rightarrow 0=m^2-4\Rightarrow m^2=4\Rightarrow m=\pm 2$. Again, since $m>0$, $m = 2$. Check that with the other equation: $y=-(2)^2+4=0$.

A third way to solve the problem would be to substitute each of the answer choices for x to find the value for which each equation equals zero. For example, starting with (A): $0=x^2-4\Rightarrow 0=(2)^2-4\Rightarrow 0=0$ and $0=-x^2+4\Rightarrow 0=-(2)^2+4\Rightarrow 0=0$.

9. **(B)** *Math: Multiple-Choice/Geometry/Volume.* The volume of a rectangular solid is equal to the area of one face multiplied by the remaining dimension. In this case, the volume is 54, so $(x)(2x)(3)=54\Rightarrow x^2=\dfrac{54}{6}=9\Rightarrow x=\sqrt{9}=3$.

10. **(C)** *Math: Multiple-Choice/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions.* The difference between the current salary, c, and the salary in y years, s, is $\dfrac{r}{2}(y)$. So, $\dfrac{r}{2}(y)$ has the same units as salary (say, dollars). Since y has units of years, r must have units of dollars per year, so the term in question is r dollars per year divided by 2, or r dollars per 2 years. Therefore, the $\dfrac{r}{2}$ term means that the employee receives a raise of r dollars every two years.

11. **(D)** *Math: Multiple-Choice/Coordinate Geometry/The Coordinate System.* Solve this item by inferring that $b>a$, so $b-a>0$. Similarly, $-c>-a$, so $c<a\Rightarrow a-c>0$. Therefore, (D) must be true.

12. **(C)** *Math: Multiple-Choice/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions.* According to the given information, $t \geq 24$ hours and $t \leq 36$ hours, so $24 \leq t \leq 36$. Based on the answer choices, try subtracting 30 from all parts of the inequality: $24 - 30 \leq t - 30 \leq 36 - 30$. Thus, $-6 \leq t - 30 \leq 6$, which is the same as the resulting equations from $|t - 30| \leq 6$ ($t - 30 \leq 6$ and $t - 30 \geq 6$). Alternatively, test the answer choices. The correct answer choice should allow only incubation times between 24 and 36 hours, inclusive. Answer choice (A) cannot be correct because $t = 36$ does not satisfy the inequality: $|36 - 30| \nless 6$. (B) is incorrect because all positive values of t satisfy the inequality, such as $t = 0$. (C) is correct because only the values between $t = 24$ and $t = 36$, inclusive, satisfy the inequality. (D) is incorrect because only negative values of t will satisfy the inequality.

13. **(A)** *Math: Multiple-Choice/Algebra/Solving Quadratic Equations and Relations* and *Geometry/Rectangles and Squares* and *Complex Figures.* The area of the border is equal to the area of the quilt with the border minus the area of the quilt without the border:
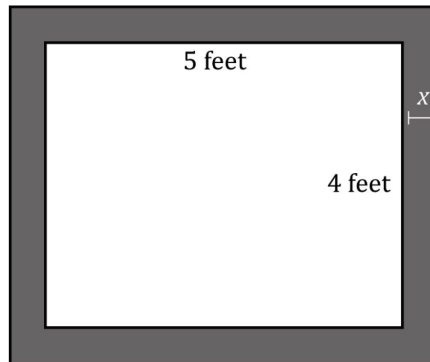

Therefore, $A_{\text{border}} = A_{\text{quilt with border}} - A_{\text{quilt}} \Rightarrow 10 = (2x + 5)(2x + 4) - (5)(4) \Rightarrow$
$10 = 4x^2 + 8x + 10x + 20 - 20 \Rightarrow 4x^2 + 18x - 10 = 0 \Rightarrow 2x^2 + 9x - 5 = 0 \Rightarrow (2x - 1)(x + 5) = 0$. Since distances can't be negative, x must be equal to $\frac{1}{2}$ foot.

14. **(B)** *Math: Multiple-Choice/Trigonometry/Determining Values on the Unit Circle.* Since $\tan x - 1 = 0$, $\tan x = 1$. And $\tan x = \frac{\sin x}{\cos x} \Rightarrow 1 = \frac{\sin x}{\cos x}$, so $\sin x = \cos x$. The item stem tells us that we are dealing with the first quadrant ($0° \leq x \leq 90°$). In the first quadrant, $\sin 45° = \frac{\sqrt{2}}{2}$ and $\cos 45° = \frac{\sqrt{2}}{2}$, thus sin$x$ and cos$x$ are equal when $x = 45°$.

15. **(D)** *Math: Multiple-Choice/Geometry/Triangles/Properties of Triangles* and *30°-60°-90° Triangles.* Complete the figure with the given information. The triangle is equilateral, so the angles are each equal to 60°. Since U and W are both midpoints, $\overline{UW}$ is parallel to $\overline{SR}$. $\triangle RST$ is similar to $\triangle WUT$ because the ratio of their sides is the same, and the angle between those sides has the same measurement. $UT{:}ST{::}WT{:}RT{::}1{:}2$ and $\angle STR = \angle UTW = 60°$. Therefore, $\angle VUT$ equals $\angle RST$: 60°. Since V is the midpoint of $\overline{UW}$ and because $\triangle TWU$ is an equilateral triangle, $\overline{VT}$ meets $\overline{UW}$ at a right angle, and $\triangle VUT$ is a 30°-60°-90° triangle.
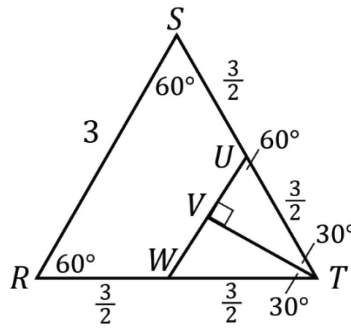
Note that each of the angles in $\triangle WUT$ is also 60°, so $\triangle WUT$ is an equilateral triangle. Thus, $\overline{UW} = \overline{UT} = \overline{WT} = \dfrac{3}{2}$ and $\overline{UV}$ is half of $\overline{UW}$, or $\dfrac{3}{4}$.

Alternatively, the length of $\overline{UV}$ can also be determined from the relationships between the sides and angles of a 30°-60°-90° triangle:
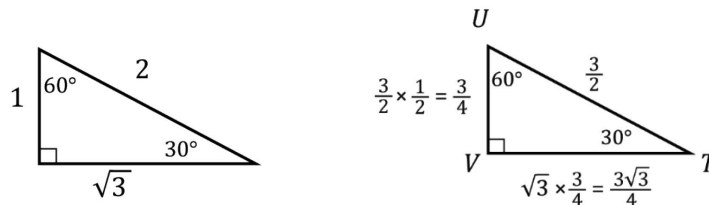
16. (72) *Math: Student-Produced Responses/Problem Solving and Advanced Arithmetic/Common Advanced Arithmetic Items.* Perform the indicated operations: $\dfrac{1}{2^{-3}} \cdot \dfrac{1}{3^{-2}} = 2^3 \cdot 3^2 = 8 \cdot 9 = 72$.

17. (3) *Math: Student-Produced Responses/Algebra/Solving Algebraic Equations or Inequalities with One Variable/Equations Involving Rational Expressions.* Solve for x^y: $\dfrac{64}{x^y} - 6 = 2 \Rightarrow \dfrac{64}{x^y} = 8 \Rightarrow x^y = \dfrac{64}{8} = 8$.

Since $8 = 2^3$, $x^y = 2^3$. Therefore, $y = 3$.

18. (24) *Math: Student-Produced Responses/Problem Solving and Advanced Arithmetic/Multi-Step Problem Solving Items.* Work backward through the problem to arrive at T: one-fourth of 16 is 4, so after one-third were turned down, 16 remained. And 16 is two-thirds of 24, so 24 people applied for jobs.

Alternatively, set up an equation: $\left(\dfrac{1}{4}\right)\left(\dfrac{2}{3}\right)T = 4 \Rightarrow \dfrac{2}{12}T = 4 \Rightarrow 2T = 48 \Rightarrow T = 24$.

19. (3) *Math: Student-Produced Responses/Algebra/Solving Simultaneous Equations.* Translate the given information into a system of simultaneous equations. Let x equal the number of groups with 3 students and y equal the number of groups with 4 students: $x + y = 12$. And since there are 45 students,

$45 = 3x + 4y$. Substitute $y = 12 - x$ for y in the second equation and solve for x:

$3x + 4y = 45 \Rightarrow 3x + 4(12 - x) = 45 \Rightarrow 3x + 48 - 4x = 45 \Rightarrow x = 3$.

20. (78) *Math: Student-Produced Responses/Geometry/Complex Figures* and *Triangles/Properties of Triangles* and *Rectangles and Squares.* Since each square is an acre, determine how many squares there are in total and how many are shaded. The entire figure is 12 squares by 9 squares, or 108 squares, and each is equal to an acre, so the entire piece of land is 108 acres. The shaded area is equal to the sum of the areas of two rectangles and two triangles:
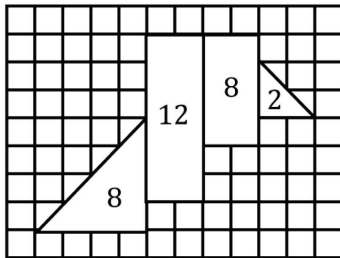
$\text{Area}_{\text{rectangle}} = \text{length} \cdot \text{width}$. $\text{Area}_{\text{triangle}} = \dfrac{\text{base} \cdot \text{height}}{2}$, or simply analyze the triangles as half-squares. Therefore, the total number of squares in the shaded area is $12 + 8 + 8 + 2 = 30$ acres. Therefore, the amount of tillable land is $108 - 30 = 78$ acres.

Section 4: Math—Calculator

1. **(A)** *Math: Multiple-Choice/Algebra/Manipulating Algebraic Expressions/Evaluating Expressions.* If the price of 5 boxes of pens is d dollars, then the price of 5 boxes of pens is $100d$ cents. Since each box contains 30 pens, the price is $100d$ cents per $5 \cdot 30 = 150$ pens, or $\dfrac{100d}{150}$. The cost of 12 pens is 12 times that, or $\dfrac{100d(12)}{150} = 8d$.

Alternatively, assume some numbers and "test-the-test" by plugging the values into the answer choices.

2. **(D)** *Math: Multiple-Choice/Problem-Solving and Advanced Arithmetic/Common Problem Solving Items/Proportions and Direct-Inverse Variation.* Set up a direct proportion and solve for the missing value: $\dfrac{\text{Cement X}}{\text{Cement Y}} = \dfrac{\text{Grit X}}{\text{Grit Y}} \Rightarrow \dfrac{4}{50} = \dfrac{20}{x} \Rightarrow 4x = (20)(50) \Rightarrow x = \dfrac{(20)(50)}{4} = 250$ cubic yards of grit.

3. **(D)** *Math: Multiple-Choice/Data Interpretation/Bar, Cumulative, and Line Graphs* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Percentages.* Add up the categories to find the total compensation: $0.8 + 0.4 + 0.6 + 0.2 = 2$ million dollars. Since wages and salaries account for 0.8 million dollars, the percentage is $\dfrac{0.8}{2} \times 100\% = 40\%$.

4. (**C**) *Math: Multiple-Choice/Problem Solving and Advanced Arithmetic/Multi-Step Problem Solving Items.* Set up an equation and solve for *J*, John's age: $\frac{2J}{3} = 12 \Rightarrow 2J = 36 \Rightarrow J = 18$.

Alternatively, test the answer choices to find the one that works. Only (C) works: (18)(2) = 36, and $36 \div 3 = 12$.

5. (**D**) *Math: Multiple-Choice/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions.* Since *y* is the cost of getting in the car plus *x* miles at $0.4 per mile, $y = 0.4x + 1.5$.

6. (**B**) *Math: Multiple-Choice/Data Interpretation/Bar, Cumulative, and Line Graphs* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Percentages.* According to the graph, the rate of rise in 1980 was 2 mm/year. In 2000, the rate of rise was 3 mm/year. Therefore, the change in rate is an increase of $\frac{3-2}{2} = \frac{1}{2} = 0.5 \times 100\% = 50\%$.

7. (**D**) *Math: Multiple-Choice/Data Interpretation/Pie Charts* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Proportions and Direct-Inverse Variation.* Begin by writing a simple sentence describing the situation: "75 is equal to 15% of the total." Set up a direct proportion or a percent problem: $\frac{75}{T} = \frac{15}{100}$, where *T* is the total number of transit vehicles. Solve for *T*:

$T = \frac{75 \times 100}{15} = 5 \times 100 = 500$.

8. (**D**) *Math: Multiple-Choice/Problem Solving and Advanced Arithmetic/Common Advanced Arithmetic Items/Properties of Numbers.* Since $\frac{x}{y}$ is an integer, and *x* and *y* are different integers they cannot both be 1, so *x* must be greater than *y*. Otherwise, *x* would not be evenly divisible by *y*. Thus, (I) is part of the correct answer. As for (II), *x* and *y* are positive integers, so *xy* > 0. Finally, *y* – *x* < 0 is equivalent to $y < x$, which is the same as (I). Therefore, (D) is the correct choice.

9. (**B**) *Math: Multiple-Choice/Statistics/Measures of Center and Spread/Median.* Median refers to the middle-most value when the numbers in a set are arranged in order. A histogram, by definition, shows the data arranged in order. Add the values and divide by two to find the median:

27 + 18 + 20 + 12 + 8 + 1 = 86 homes included, and $86 \div 2 = 43$, so the 43rd home is the median. Now count from the left to find the 43rd home. The 43rd home has one child.

Number of Children per Home
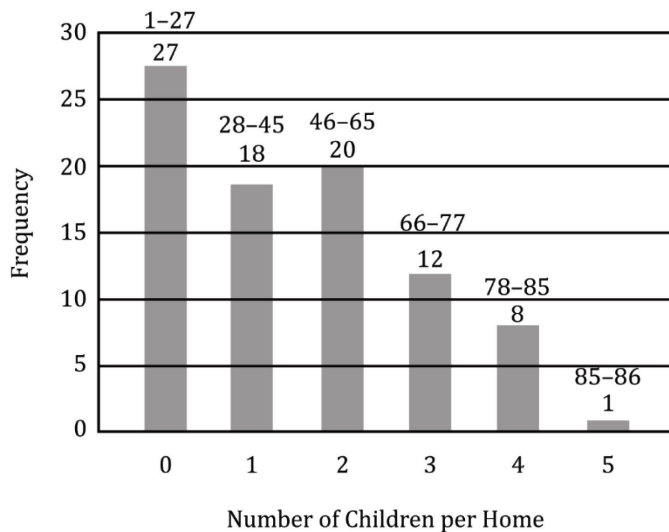
10. **(C)** *Math: Multiple-Choice/Statistics/Measures of Center and Spread/Averages.* Create an expression for a weighted average: $\dfrac{\text{total \# of children}}{\text{total \# of homes}} = \dfrac{27(0)+18(1)+20(2)+12(3)+8(4)+1(5)}{27+18+20+12+8+1} = \dfrac{131}{86} \approx 1.5$.

11. **(C)** *Math: Multiple-Choice/Statistics/Data Interpretation* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Proportions and Direct-Inverse Variation.* The total number of homes in the represented neighborhood is $27+18+20+12+8+1=86$, 27 of which are childless. The item stem states that this proportion is true for the entire city, so $\dfrac{27}{86} = \dfrac{x}{3.5\times10^5}$. Solve for x:

$x = \dfrac{27\times3.5}{86}\times10^5 \approx 1.1\times10^5 = 110{,}000$ homes in the entire city are childless.

12. **(B)** *Math: Multiple-Choice/Algebra/Solving Simultaneous Equations.* Let x, y, and z represent the number of each type of vegetable. The total garden area is 12 square feet, so

$x \text{ plants}\times\dfrac{0.5 \text{ square foot}}{\text{X plant}} + y \text{ plants}\times\dfrac{0.5 \text{ square foot}}{\text{Y plant}} + z \text{ plants}\times\dfrac{2 \text{ square feet}}{\text{Z plant}} = 12$ square feet:

$\dfrac{1}{2}(x+y)+2z = 12$. Since $x = y = z$, $\dfrac{1}{2}(2x)+2x = 12 \Rightarrow 3x = 12 \Rightarrow x = 4$. Therefore, there are four of each type of vegetable, for a total of 12 plants.

13. **(A)** *Math: Multiple-Choice/Problem-Solving and Advanced Arithmetic/Multi-Step Problem Solving Items.* The time spent traveling is equal to the distance divided by the rate. In this case, the total time spent traveling is $\dfrac{10 \text{ miles}}{20 \text{ miles/hour}} + \dfrac{10 \text{ miles}}{30 \text{ miles/hour}} + \dfrac{10 \text{ miles}}{60 \text{ miles/hour}}$. Give the fractions a common denominator, so the total time is $\dfrac{3}{6}+\dfrac{2}{6}+\dfrac{1}{6}=\dfrac{6}{6}=1$ hour. The motorist spent $\dfrac{1}{6}$ hour of the total driving time of 1 hour driving 60 miles per hour, so $\dfrac{1}{6}$ of her total driving time was at that rate.

14. (**A**) *Math: Multiple-Choice/Data Interpretation/Scatterplots.* The line of best fit is a straight line that best represents the data on a scatterplot. It's easy to see, with an estimate of the line of best fit added, that the data point farthest from the line is for the 10 mph (68°F) data point:

Weather Station Readings, Noon
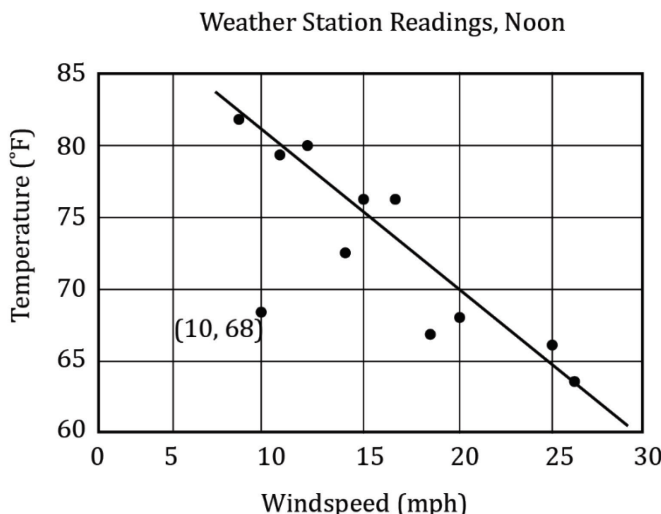
15. (**C**) *Math: Multiple-Choice/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions.* Translate the information a step at a time, including units. If x represents the number of haircuts, the charge for x cuts is $\dfrac{\$22}{\text{haircut}} \times x \text{ haircuts} = 22x$, in dollars. The charge for y additional minutes of styling for half the haircuts, or $\dfrac{x}{2}$, is

$\dfrac{\$3}{10 \text{ additional minutes}} \times \dfrac{y \text{ additional minutes}}{\text{haircut}} \times \dfrac{x}{2} \text{ haircuts} = \dfrac{3xy}{20}$, in dollars. The total charges is sum

of the two expressions: $22x + \dfrac{3xy}{20}$.

16. (**D**) *Math: Multiple-Choice/Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Proportions and Direct-Inverse Variation.* Using the ideal gas law $pV = nRT$, since n, R, and T are all constant, $p_1V_1 = p_2V_2$. Fill in the supplied values: $p_1V_1 = p_2V_2 \Rightarrow (3 \text{ atm})(1 \text{ L}) = p_2\left(\dfrac{1}{3} \text{ L}\right) \Rightarrow$

$p_2 = \dfrac{(3 \text{ atm})(1 \text{ L})}{\dfrac{1}{3} \text{ L}} = 9 \text{ atm}$.

17. (**D**) *Math: Multiple-Choice/Data Interpretation/Scatterplots.* The scatterplot shows a positive association between x and y; that is, as x increases, y increases. (B) and (C) are incorrect because they both indicate a negative relationship. (A) is incorrect because there is a clear trend in the scatterplot.

18. (**A**) *Math: Multiple-Choice/Data Interpretation/Tables.* The following information is implicit in the table:

FORTUNE 500 COMPANIES, 2013			
	Senior Level Executives	**CEOs**	**Total**
Women	8,785		
Men	26,215		
Total	35,000	500	35,500

So, (A) is already known. Any of the other four numbers would allow for the completion of the table.

19. (**C**) *Math: Multiple-Choice/Data Interpretation/Tables* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Percentages.* Use the given information to fill in the table:

$0.048 \times 500 = 24$.

| Fortune 500 COMPANIES, 2013			
	Senior Level Executives	**CEOs**	**Total**
Women	8,785	24	8,809
Men	26,215	476	26,691
Total	35,000	500	35,500

Therefore, the percentage of CEO and senior-level executive positions combined at Fortune 500 companies held by men is $\dfrac{26{,}691}{35{,}500} \approx 75\%$.

20. (**D**) *Math: Multiple-Choice/Data Interpretation/Tables* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/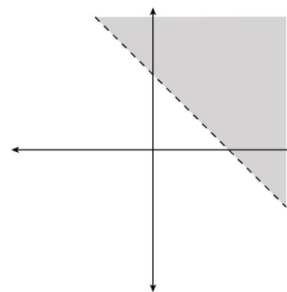Proportions and Direct-Inverse Variation.* Based on the given information, approximately $\dfrac{24}{500} = \dfrac{x}{35 \text{ million}} \Rightarrow x = \dfrac{24(35 \text{ million})}{500} = 1.68$ million.

21. (**A**) *Math: Multiple-Choice/Algebra/Solving Simultaneous Equations.* The possible values for *y* in each inequality are shown below:
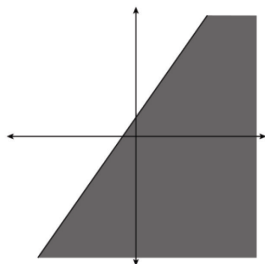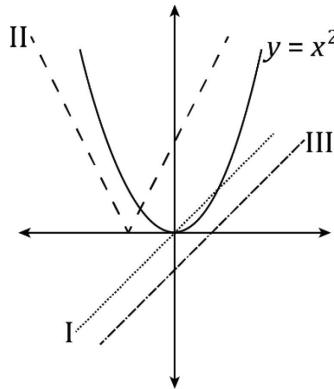
$$y \le \frac{1}{2} + \frac{3}{2}x \qquad\qquad\qquad\qquad y > -x + 2$$

Therefore, the solutions that are common to both inequalities are in Section II of the graph shown in the item stem.

22. (**B**) *Math: Multiple-Choice/Coordinate Geometry/Graphs of Linear Equations* and *Graphs of Quadratic Equations and Relations.* The quickest solution to this item is to quickly sketch the four equations:

The only equation that does not intersect the equation in the stem is (III). Therefore, (B) is the correct choice.

Note that this item can also be solved using the system of simultaneous equations (by setting each of the three equations equal to the stem equation and determining if there are values for *x* that make the equality true). However, a sketch of the equations is a much faster solution.

23. (**D**) *Math: Multiple-Choice/Geometry/Triangles/Properties of Triangles* and *30°-60°-90° Triangles.* This problem can be solved with or without trigonometry. First, with trigonometry: ΔPQR is equilateral and therefore equiangular with three 60° angles. And $\angle PQS$ is 30°. Since ΔPQS is a 30°-60°-90° triangle, it has special properties: PQ = 6 and $QS = 3\sqrt{3}$. Therefore, the altitude $\overline{QS}$ has a length of $3\sqrt{3}$ and the base $\overline{PR}$ has a length of (2)(3) = 6, so the area is $\frac{1}{2}(3\sqrt{3})(6) = 9\sqrt{3}$, which is approximately 15.58.

Note that the cosine relationship can also be used to find the length of $\overline{QS}$: $\cos\angle PQS = \cos 30 = \dfrac{\overline{QS}}{PQ} \Rightarrow$

$\dfrac{\sqrt{3}}{2} = \dfrac{QS}{6} \Rightarrow QS = \dfrac{6\sqrt{3}}{2} = 3\sqrt{3}$. And again, the area is: $\frac{1}{2}(3\sqrt{3})(6) = 9\sqrt{3}$.

Alternatively, solve the problem without trigonometry: $\Delta PQS \cong \Delta RQS$, $\overline{PS} = \overline{SR}$, and $\overline{PQ} = \overline{PR} \Rightarrow$ $PQ = 2PS$. Since PS = 3, PQ = 6. ΔPQS is a right triangle, so by the Pythagorean theorem: $(PQ)^2 = (PS)^2 + (QS)^2 \Rightarrow 6^2 = 3^2 + (QS)^2 \Rightarrow (QS)^2 = 36 - 9 = 27 \Rightarrow QS = \sqrt{27} = 3\sqrt{3}$. Then, $\text{area} = \frac{1}{2} \cdot 6 \cdot 3\sqrt{3} = 9\sqrt{3}$.

24. (**B**) *Math: Multiple-Choice/Algebra/Manipulating Algebraic Expressions/Evaluating Expressions* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Ratios.* Using the rule for

constructing the sequence, if n is the first term, then, the second term is equal to $\frac{1}{2}(n+4) = \frac{n+4}{2}$. The ratio of the first term, n, to the second term, $\frac{n+4}{2}$, is $\frac{n}{\frac{n+4}{2}} = n \cdot \frac{2}{n+4} = \frac{2n}{n+4}$.

25. (C) *Math: Multiple-Choice/Data Interpretation/Scatterplots.* Simply read the graph to find the force value (y-axis) that corresponds to the data point recorded for a distance (x-axis) of 0.5 meters: 50 newtons.

26. (B) *Math: Multiple-Choice/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions/Functions as Models.* Plug the given values into the function:

$$F = \left(\frac{50 \text{ newtons}}{\text{meter}}\right)(0.25 \text{ meter}) = 12.5 \text{ newtons.}$$

27. (C) *Math: Multiple-Choice/Coordinate Geometry/Slope-Intercept Form of a Linear Equation* and *Graphs of Linear Equations* and *Data Interpretation/Scatterplots.* For the linear equation $F = kx$, k is the slope of the line when F is graphed as a function of x. Approximate a line of best fit through the data:
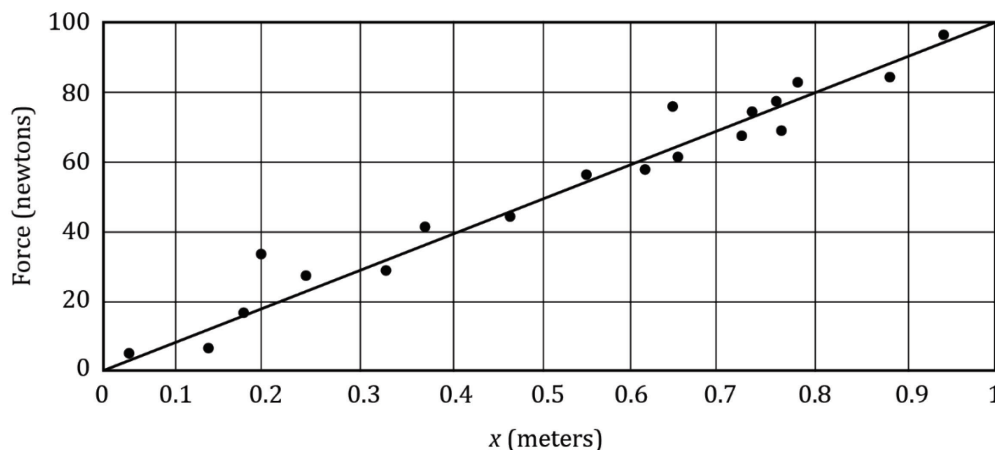
The slope of the line is the change in y divided by the change in x, or approximately $\frac{100-0}{1-0} = 100$ newtons per meter. Convert this to dynes per centimeter:

$$\frac{100 \text{ newtons}}{\text{meter}} \times \frac{1 \text{ meter}}{100 \text{ centimeters}} \times \frac{1 \text{ dyne}}{10 \text{ micronewtons}} \times \frac{10^6 \text{ micronewtons}}{1 \text{ newtons}} = 10^5 \frac{\text{dynes}}{\text{centimeter}}.$$

28. (C) *Math: Multiple-Choice/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions.* The number of eggs in Basket A after transfer of n eggs from Basket B is $a = n+6$; the number of eggs in Basket B after transfer of n eggs to Basket A is $b = 24-n$. The final number of eggs in basket B, b, is at least twice the final number of eggs in Basket A, a, so $2a \le b$.

29. (C) *Math: Multiple-Choice/Coordinate Geometry/Slope-Intercept Form of a Linear Equation.* Begin by finding the slope from the two points on the line, $(-1,0)$ and $(2,2)$: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{2-(-1)} = \frac{2}{3}$. b is the y

value when the line crosses the y-axis (where $x = 0$). The equation of the line is $y = mx + b$; m is $\frac{2}{3}$. Plug in a known point on the line and solve for b. $0 = \frac{2}{3}(-1) + b \Rightarrow b = \frac{2}{3}$. Therefore: $y = \frac{2x}{3} + \frac{2}{3}$.

30. **(D)** *Math: Multiple-Choice/Algebra/Solving Simultaneous Equations.* The given inequality is actually a system of two inequalities: $-6 \leq -6(p-5)$ and $-6(p-5) \leq 0$. Solve each for p. First, $-6 \leq -6(p-5) \Rightarrow -6 \leq -6p + 30 \Rightarrow -36 \leq -6p \Rightarrow 6 \geq p$. Notice that in the last step the inequality sign reversed direction due to division by a negative number. Second, $-6(p-5) \leq 0 \Rightarrow -6p + 30 \leq 0 \Rightarrow -6p \leq -30 \Rightarrow p \geq 5$. Combine the two ranges for p into one expression: $5 \leq p \leq 6$, (D).

31. **(27)** *Math: Student-Produced Responses/Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Percentages* and *Data Interpretation/Bar, Cumulative, and Line Graphs.* Since all of the data are reported as rates (the number of crimes per 100,000 people), simply deal with the number of crimes. In 1991, there were 11,000 total crimes and in 2006, there were 8,000 total crimes—a decrease of 3,000 crimes. The total crime decrease is equal to $\frac{3,000 \text{ total crimes}}{11,000 \text{ total crimes}} \approx 0.27 = 27\%$ of the total crimes in 1991.

32. **(5)** *Math: Student-Produced Responses/Statistics/Measures of Center and Spread/Averages* and *Data Interpretation/Bar, Cumulative, and Line Graphs.* The number of hurricanes in categories 3, 4, and 5 for each decade is the difference between the total bar length and the number of hurricanes in categories 1 and 2: $12 - 8 = 4$, $15 - 11 = 4$, $14 - 9 = 5$, and $19 - 12 = 7$. Therefore, the average number of hurricanes in categories 3, 4, and 5 hitting the US mainland per decade since 1971 is $\frac{4+4+5+7}{4} = \frac{20}{4} = 5$.

33. **(2/3, 4/6, 0.66)** *Math: Student-Produced Responses/Algebra/Solving Quadratic Equations and Relations.* If you remember that for a quadratic equation in the form $ax^2 + bx + c = 0$, the sum of the roots of the equation is equal to $-\frac{b}{a}$ (the product is equal to $\frac{c}{a}$), this problem is quick to solve. In the given equation, $a = 3$, $b = -2$, and $c = -16$, so the sum of the roots is $-\frac{b}{a} = -\frac{-2}{3} = \frac{2}{3}$.

Alternatively, factor the equation: $3x^2 - 2x - 16 = 0 \Rightarrow (3x - 8)(x + 2) = 0$. Set each factor equal to 0 and solve: $(3x - 8) = 0 \Rightarrow x = \frac{8}{3}$ and $(x + 2) = 0 \Rightarrow x = -2$. And the sum of the roots is

$\frac{8}{3} + (-2) = \frac{16}{6} + \frac{-12}{6} = \frac{4}{6} = \frac{2}{3}$.

34. **(20)** *Math: Student-Produced Responses/Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Percentages* and *Geometry/Volume.* Use the equation for the volume of a cylinder, $V = \pi r^2 h$, where r is the radius and h is the height. If the overall volume increase must be at least 25%,

$V' = \frac{5}{4}V = \frac{5}{4}(\pi r^2 h) = \frac{5\pi h}{4}\left(\frac{d}{2}\right)^2 = \frac{5\pi h}{16}d^2$. Since the new diameter is to be increased by 50%, $d' = \frac{3}{2}d$, so

$V' = \frac{\pi h'}{4}d'^2 = \frac{\pi h'}{4}\left(\frac{3}{2}d\right)^2 = \frac{9\pi h'}{16}d^2$. Set the two expressions equal to one another and solve for h':

$\frac{5\pi h}{16}d^2 = \frac{9\pi h'}{16}d^2 \Rightarrow 5h = 9h' \Rightarrow h' = \frac{5}{9}h$. Since $h = 36$ feet, the new height must be at least $\frac{5}{9}(36) = 20$ feet.

35. (7) *Math: Student-Produced Responses/Algebra/Solving Algebraic Equations or Inequalities with One Variable/Equations Involving Exponents.* Rewrite the equation so both sides have the same base, 3:

$3^{n-4} = 27 \Rightarrow 3^{n-4} = 3^3$. Set the exponents equal and solve for n: $n - 4 = 3 \Rightarrow n = 7$.

36. (75) *Math: Student-Produced Responses/Geometry/Circles* and *Problem Solving and Advanced Arithmetic/Common Problem Solving Items/Proportions and Direct-Inverse Variation.* There are 360 degrees in a circle, so 8 hours is represented by 360 degrees. Set up a direct proportion to determine the size of the sector representing 100 minutes (20 minutes + 20 minutes + 60 minutes):

$\frac{100 \text{ minutes}}{8 \times 60 \text{ minutes}} = \frac{x°}{360°} \Rightarrow x = \frac{100 \times 360}{8 \times 60} = \frac{100 \times 6}{8} = \frac{100 \times 3}{4} = 75°$.

37. (1) *Math: Student-Produced Responses/Problem Solving and Advanced Arithmetic/Common Advanced Arithmetic Items/Complex Numbers.* Rationalize the denominator in the fraction by multiplying the numerator and the denominator by the conjugate of the denominator (a complex number multiplied by its conjugate results in a real number). Expand the binomials using the FOIL method for multiplying polynomials: $\frac{4+2i}{3-i} \cdot \frac{3+i}{3+i} = \frac{(4+2i)(3+i)}{(3-i)(3+i)} = \frac{12+4i+6i+2i^2}{9+3i-3i-i^2} = \frac{12+10i+2i^2}{9-i^2}$. Since $i = \sqrt{-1}$, $i^2 = -1$, so

substitute and simplify: $\frac{12+10i+2(-1)}{9-(-1)} = \frac{10+10i}{10} = 1+i$. Therefore, $1+i = a+bi$, so $a = 1$.

38. (4) *Math: Student-Produced Responses/Algebra/Creating, Expressing, and Evaluating Algebraic Equations and Functions* and *Solving Simultaneous Equations.* Create an equation for the total cost under each special as a function of the number of games per bowler. In the first case,

$\frac{\$5}{\text{game}} \times x \text{ games} \times 4 \text{ players} = \$20x$ and a discount of $20 for more than 3 games per bowler. Thus, for $x \leq 3$, $C = 20x$; for $x > 3$, $C = 20x - 20$. In the second case,

$\frac{\$3}{\text{shoes}} \times 4 \text{ players} + \frac{\$3}{\text{game}} \times 4 \text{ players} \times x \text{ games} = 12 + 12x$. For each range of x (integers less than or equal to 3 and greater than 3), set the equation for the first special equal to the equation for the second special and solve for x games. For $x \leq 3$, $20x = 12x + 12 \Rightarrow 8x = 12 \Rightarrow x = \frac{3}{2}$; for $x > 3$,

$20x - 20 = 12x + 12 \Rightarrow 8x = 32 \Rightarrow x = 4$. Since purchasing 1.5 games per bowler is not possible, the correct answer is 4.

Section 5: Essay

Sample Essay Responses and Analyses

Above Average Response

Although we no longer have segregated schools, an argument has been made that educational segregation is "the civil rights issue of our generation." In his speech "Sixty Years After Brown: Where is the Outrage?" former Secretary of Education Arne Duncan makes this exact argument, presenting facts and statistics that show certain groups still do not have the same access to high-quality education that white students have. Duncan discusses that, even though we are no longer under the Jim Crow school segregation laws, equal opportunity is still not a reality for many minority, female, disabled, LGBT and disadvantaged students.

Duncan begins his argument by sharing some statistics about modern school segregation. According to the information he presents in his speech, all regions in the US have seen an increased number of African American students attending highly segregated schools (that is, a school where 90 percent or more of the students are students of color). Duncan clarifies that this type of segregation is not by law (which is now unconstitutional after the Brown v. Board of Education decision), but rather by fact. The South sees more than a third of African American students attending this type of school. Duncan includes these statistics to demonstrate the status of segregation in the United States today, particularly to show that more African Americans are "attending highly segregated schools," and to startle the audience by demonstrating that this development toward inequality in education continues despite segregation being officially outlawed. These statistics successfully serve their function because they debunk audience's expectation that, following the Brown v. Board of Education decision, segregation would decrease. Duncan demonstrates that the very opposite has occurred, shocking the audience.

Another point Duncan makes to persuade his audience into considering education a civil rights issue is the disparity in access to advanced learning courses (AP courses, calculus, etc.). He states that only 68 percent of African American students have access to calculus courses that are readily available to 81 percent of their white and 87 percent of their Asian counterparts. Not all families have access to high-quality early learning (pre-kindergarten), which places their children at a disadvantage when it comes time to begin schooling. Duncan includes these statistics in order to go beyond the inequality in racial composition between schools and to show that there are even disparities in students' access to a quality curriculum. The fact that less than 70% of African American students can access calculus courses, as opposed to over 80% of white and Asian students, is meant to outrage the audience. It serves its purpose effectively because it refutes the listener's presumed expectation that access to AP courses is equal across races in a post-segregation society.

Duncan also builds his argument by identifying instances of real-world inequality in education for particular groups. He cites the reduction of the grades of students with disabilities in a New York county by .69 and the withholding of AP courses at predominantly-minority Alabama high schools being justified by the incapability of black students to succeed in the coursework. These examples are meant to show that unequal access to educational opportunity is a tangible, ongoing reality. They reinforce that the statistics Duncan cites throughout refer to actual populations and are not just empty, exaggerated numbers. They work because people can connect to other people in a way they cannot connect to numbers; one finds him/herself empathizing with the New York and Alabama students and desiring justice for them and for other neglected groups. The people affected by the opportunity gap become comprehendible, relatable human beings to the reader, and in this way, the examples evoke reader empathy and support for Duncan's claim.

While we tend to think that school segregation is an issue of the past, namely because de jure segregation no longer exists, Duncan reminds us that educational segregation is quite possibly the biggest civil rights issue of our time. He is able to make this compelling argument by discussing facts and statistics that clearly show a lack

of access to educational opportunities for African American students. He also discusses actual examples of educational inequality that have affected African American students as well as other groups. Combined, these persuasive strategies successfully demonstrate that education is a critical civil rights issue for this generation.

Analysis of Above Average Response

The writer demonstrates an understanding of the issues presented in the passage, and he/she effectively discusses how the speaker made his argument. The use of textual evidence makes the writer's discussion logical and compelling and shows a solid grasp on the speaker's claims. The writer's choice of vocabulary and sentence structure makes for a response with an appropriate style.

Below Average Response

Since there is no more segregated schools, you would think that we don't have to worry about black people being treated unfairly when it comes to education. But really that is not the case. Even in the modern age there are problems with black students not having as good of an education as white students. This is what Arne Duncan talks about in his speech, the way that education is still segregated in a way, even though there aren't just black schools and white schools. In this essay I am going to talk about how education is still kind of segregated and why according to Arne Duncan's speech.

The first reason Arne Duncan talks about is the fact that black student attend schools with mostly other black students. This means that they don't have access to alot of the classes that white students have. They also don't have the same opportunities as white students. The reason why this happens is because they can't figure out why there are so many black students all going to the same schools and why it is not more balanced. It is not because of the law because the old laws are gone and you can't make black students go to one school and white students go to another. For some reason this happens and it causes segregation.

The second reason why there is segregation still today is because it includes more than just black students. Female students, disabled students, LGBT students, and students who are still learning English. These students also are sort of separated from the typical white students that have all the educational opportunities open to them without any problem. There needs to be a way to make sure that all students can have access to all the educational opportunities that white students have. This would help with the segregation.

The final reason why schools are still segregated is that there is still discrimination. People like to think that discrimination ended a long time ago but you can still see it alot of places today. One of the places you find it is in schools. It really is true that there is still discrimination and we need to figure out a way to stop it.

All in all it is surprising that discrimination still exists today. Schools are actually the one place where there might be the most segregation, because there are lots of students and not just black students who are not getting the same education as white students. I think Arne Duncan was right when he talked about this and we need to figure out a way to make sure this segregation and discrimination ends.

Analysis of Below Average Response

While the response follows a somewhat logical sequence, it is clear that the writer does not have a firm grasp on the topic presented in the passage. There are issues of subject-verb agreement and word choice that make the essay difficult to read. The writer does not use much evidence from the text to support his/her argument, and some of the information in the essay may not be accurate. The writer missed the point of the prompt and did not discuss how the speaker developed his argument.