

Math Express Summary

Arithmetic

1. Natural Numbers: *Natural* numbers are the set of positive integers and are also referred to as counting numbers: $\{1, 2, 3, 4, 5, \dots\}$.

2. Whole Numbers: *Whole* numbers are the numbers used for counting, plus the number zero: $\{0, 1, 2, 3, 4, 5, \dots\}$.

3. Integers: *Integers* are positive or negative whole numbers.

e.g., $-568, -45, 0, 6, 67, \frac{16}{2}, 345$

4. Positive and Negative Integers: When adding integers that have the *same* sign, add their values with positive signs, and give the result the same sign as the original values.

e.g., 1. $4 + 8 = 12$

2. $-5 + (-3) = -(5 + 3) = -8$

When adding integers that have *opposite* signs, take their values with positive signs, subtract the smaller from the larger, and then give the result the sign of the integer with the larger positive value.

e.g., 1. $-3 + 2 = -(3 - 2) = -1$

2. $4 + (-6) = -(6 - 4) = -2$

Subtracting an integer of any sign from another integer of any sign is the same as adding its opposite.

e.g., 1. $4 - 8 = 4 + (-8) = -4$

2. $-3 - 2 = -3 + (-2) = -5$

3. $-5 - (-3) = -5 + 3 = -2$

4. $7 - (-4) = 7 + 4 = 11$

5. Even and Odd Integers: An *even* integer is evenly divisible by 2, whereas an *odd* integer is not evenly divisible by 2. 0 is an even integer.

e.g., 1. $-50, -4, 0, 2, 34$ are even integers.

2. $-45, -3, 9, 15$ are odd integers.

Important properties of even and odd integers:

even + even = even	e.g., $2 + 4 = 6$
even + odd = odd	e.g., $4 + 3 = 7$
odd + odd = even	e.g., $3 + 5 = 8$
odd + even = odd	e.g., $3 + 4 = 7$
even • even = even	e.g., $2 \cdot 4 = 8$
even • odd = even	e.g., $2 \cdot 3 = 6$
odd • odd = odd	e.g., $3 \cdot 5 = 15$
odd • even = even	e.g., $3 \cdot 2 = 6$

6. Consecutive Integers: *Consecutive* integers are in continuous sequence. If the first integer of a consecutive sequence is m , the sequence is $m, m + 1, m + 2$, etc.

e.g., 1. $\{4, 5, 6, 7, \dots\}$

2. $\{-10, -9, -8, -7, \dots\}$

Consecutive *even* or *odd* integers are in continuous sequence of even or odd integers, respectively. An even or odd sequence is $m, m + 2, m + 4, m + 6$, etc.

e.g., 1. $\{-4, -2, 0, 2, \dots\}$

2. $\{7, 9, 11, 13, \dots\}$

7. Real Numbers: Real numbers are all the numbers on the number line, including integers, decimals, fractions, and radical numbers.

e.g., $-\frac{1}{2}, 0, \frac{2}{3}, \sqrt{2}, \pi$

A real number is *rational* if it can be written as the ratio of two integers, where the denominator does not equal zero. Natural numbers, whole

numbers, integers, common fractions, and repeating decimals are examples of rational numbers.

e.g., $-\frac{1}{2}$, 0, $0.\overline{11}$, 0.75, $\frac{2}{3}$

A real number is *irrational* if it cannot be written as the ratio of two integers. Irrational numbers have infinite non-repeating decimal representations.

e.g., $\sqrt{2}$, π

Properties of real numbers:

$(+)(+) = (+)$

$(-)(-) = (+)$

$(+)(-) = (-)$

$(-)^2 = (+)$

$m + 0 = m$, where m is a real number

$m \cdot 0 = 0$, where m is a real number

e.g., 1. $\left(\frac{1}{2}\right)(4) = 2$

2. $(-2)\left(-\frac{4}{5}\right) = \frac{8}{5}$

3. $(2)(-4) = -8$

4. $\left(-\frac{3}{4}\right)(4) = -3$

5. $(-2)^2 = 4$

6. $2 + 0 = 2$

7. $(2)(0) = 0$

- 8. Factor:** A *factor* is a number that divides evenly into another number.

e.g., 1, 2, 3, 4, 6, and 12 are factors of 12

- 9. Prime:** A *prime* number is any natural number (except 1) that is divisible only by 1 and itself.

e.g., 2, 3, 5, 7, 11, 13, 17, and 19 are all prime numbers

- 10. Prime Factors:** All natural numbers can be expressed as the product of prime numbers, which are called the *prime factors* of that number.

e.g., 1. $3 = (3)(1)$

2. $12 = (2)(2)(3)$

- 11. Miscellaneous Symbols:**

“=” means “is equal to”

“≠” means “is not equal to”

“<” means “is less than”

“>” means “is greater than”

“≤” means “is less than or equal to”

“≥” means “is greater than or equal to”

“|” means “absolute value (always non-negative)”

e.g., 1. $3 = 3$

2. $\frac{3}{4} \neq \frac{5}{6}$

3. $-3 < 6$

4. $5 > 4$

5. $m - 3 \leq -3$,
for $m = \dots, -3, -2, 0$

6. $m + 3 \geq 3$,
for $m = 0, 1, 2, 3, \dots$

7. $|-5| = 5$

- 12. Terms:** The *sum* or *total* is the result of adding numbers together. The *difference* is the result of subtracting one number from another. The *product* is the result of multiplying numbers together. The *quotient* is the result of dividing one number by another. The *remainder* is the number remaining after one number is divided into another number.

e.g., 1. The sum (or total) of 2 and 3 is 5:
 $2 + 3 = 5$.

2. The difference between 5 and 2 is 3:
 $5 - 2 = 3$.

3. The product of 2 and 3 is 6: $(2)(3) = 6$.



4. The quotient of 6 divided by 2 is 3:
 $6 \div 2 = 3$.

5. The remainder of 7 divided by 3 is 1:
 $7 \div 3 = 2$ plus a remainder of 1.

13. Fractions: When one whole integer is divided by another whole integer (other than zero) and the result is not a third whole integer, the result is a fraction, or ratio. The top number is called the *numerator*; the bottom number is called the *denominator*.

e.g., 2 divided by 3 results in a fraction, not a whole number: $2 \div 3 = \frac{2}{3}$.

Proper fractions have a numerator of lower value than the denominator and thus have a value less than 1.

e.g., $\frac{1}{2}$ and $\frac{3}{4}$ are both less than 1.

Improper fractions have a numerator of greater value than the denominator, and thus have a value greater than 1.

e.g., $\frac{3}{2}$ and $\frac{4}{3}$ are both greater than 1.

A *mixed number* consists of both a whole number and a fraction written together.

e.g., 1. $2\frac{1}{2} = 2 + \frac{1}{2}$

2. $3\frac{4}{5} = 3 + \frac{4}{5}$

To add, subtract, multiply, or divide fractions, convert mixed numbers to improper fractions as follows:

a) The new denominator is the denominator of the fractional part of the mixed number.

b) The new numerator is the whole number of the mixed number multiplied by the denominator of the fractional part and then added to its numerator.

e.g., 1. $3\frac{1}{4} = \frac{(3 \cdot 4) + 1}{4} = \frac{13}{4}$

2. $6\frac{2}{5} = \frac{(6 \cdot 5) + 2}{5} = \frac{32}{5}$

3. $2\frac{12}{13} = \frac{(2 \cdot 13) + 12}{13} = \frac{38}{13}$

To convert an improper fraction to a mixed number, reverse the process as follows:

a) Divide the denominator into the numerator. The integer part of the quotient becomes the whole number part of the mixed number.

b) With the same denominator, create a fraction with the numerator equal to the remainder of the first step.

e.g., 1. $\frac{29}{5} = 29 \div 5$
 $= 5$ with a remainder of 4
 $= 5\frac{4}{5}$

2. $\frac{31}{6} = 31 \div 6$
 $= 5$ with a remainder of 1
 $= 5\frac{1}{6}$

3. $\frac{43}{13} = 43 \div 13$
 $= 3$ with a remainder of 4
 $= 3\frac{4}{13}$

14. Reducing Fractions: It is conventional to reduce all fractions to lowest terms. To reduce a fraction to lowest terms, eliminate redundant factors that are in both the numerator and the denominator. Either factor or divide out the redundant factors from both.

e.g., $\frac{8}{16} = \frac{1(8)}{2(8)} = \frac{1}{2}$, or $\frac{8}{16} = \frac{8 \div 8}{16 \div 8} = \frac{1}{2}$

A fraction is expressed in *lowest terms* when there is no natural number (other than 1) that can

be divided evenly into both the numerator and the denominator.

e.g., $\frac{8}{15}$ is in lowest terms, as there is no natural number (other than 1) that divides evenly into 8 and 15.

15. Common Denominators: A *common denominator* is a number that is a multiple of the denominators of two or more fractions.

e.g., Since 12 is an even multiple of both 3 and 4, it is a common denominator for $\frac{1}{3}$ and $\frac{1}{4}$.

Converting a fraction to another denominator is the reverse of reducing it to lowest terms. Multiplying the numerator and the denominator of a fraction by the same number is equal to multiplying it by 1, which means that the value is unchanged.

$$\text{e.g., 1. } \frac{1}{4} = \frac{(1)(3)}{(4)(3)} = \frac{3}{12}$$

$$2. \frac{2}{3} = \frac{(2)(4)}{(3)(4)} = \frac{8}{12}$$

16. Complex Fractions: A *complex fraction* is a fraction in which either the numerator or the denominator, or both, contains fractions. There are two methods for simplifying complex fractions.

Method 1: Multiply the numerator by the reciprocal of the denominator and simplify.

$$\text{e.g., } \frac{\frac{1}{2}}{\frac{3}{4}} = \left(\frac{1}{2}\right)\left(\frac{4}{3}\right) = \frac{4}{6} = \frac{2}{3}$$

Method 2: Multiply both the numerator and the denominator by the least common denominator for the terms in the numerator and the denominator of the complex fraction and simplify.

$$\text{e.g., } \frac{\frac{1}{2}}{\frac{3}{4}} \cdot \frac{4}{4} = \frac{\frac{4}{2}}{\frac{12}{4}} = \frac{2}{3}$$

17. Adding Fractions: The procedure for adding fractions varies depending on whether or not the fractions already share the same denominator.

To add fractions with the same denominator, create a new fraction using the common denominator. The new numerator is the sum of the old numerators.

$$\text{e.g., } \frac{3}{7} + \frac{2}{7} = \frac{5}{7}$$

To add fractions with different denominators, find a common denominator and convert the fractions.

$$\text{e.g., 1. } \frac{1}{3} + \frac{1}{5} = \frac{1(5)}{3(5)} + \frac{1(3)}{5(3)} = \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$$

$$2. \frac{1}{3} + \frac{2}{7} = \frac{1(7)}{3(7)} + \frac{2(3)}{7(3)} = \frac{7}{21} + \frac{6}{21} = \frac{13}{21}$$

To add a fraction and a whole number, treat the whole number as a fraction with a denominator of 1.

$$\begin{aligned} \text{e.g., } 2 + \frac{1}{5} + \frac{1}{2} &= \frac{2}{1} + \frac{1}{5} + \frac{1}{2} \\ &= \frac{2(10)}{1(10)} + \frac{1(2)}{5(2)} + \frac{1(5)}{2(5)} \\ &= \frac{20}{10} + \frac{2}{10} + \frac{5}{10} \\ &= \frac{27}{10} \end{aligned}$$

To add a fraction and a mixed number, change the mixed number to an improper fraction and then add.

$$\text{e.g., } 2\frac{1}{3} + \frac{1}{3} = \frac{7}{3} + \frac{1}{3} = \frac{8}{3} = 2\frac{2}{3}$$

18. Subtracting Fractions: Follow the same procedure for addition, except subtract rather than add.

To subtract fractions with the same denominator, simply subtract the second numerator from the first.

$$\text{e.g., } \frac{5}{7} - \frac{2}{7} = \frac{3}{7}$$



To subtract fractions with different denominators, first find a common denominator.

$$\text{e.g., } \frac{7}{8} - \frac{3}{5} = \frac{7(5)}{8(5)} - \frac{3(8)}{5(8)} = \frac{35}{40} - \frac{24}{40} = \frac{11}{40}$$

- 19. “Flying-X” Method for Adding and Subtracting Fractions:** It is not necessary to find the least common denominator when adding or subtracting fractions if you reduce the result to lowest terms. Any common denominator will work—simply use the “flying-x” method.

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \xrightarrow{\text{+}} \frac{c}{d} = \frac{ad + bc}{bd}$$

Step 1: Multiply the denominators together to get a new denominator.

Step 2: Multiply the numerator of the first fraction by the denominator of the second.

Step 3: Multiply the denominator of the first fraction by the numerator of the second.

Step 4: The new numerator is the sum (or difference) of the results of steps 2 and 3.

$$\text{e.g., } 1. \frac{2}{7} + \frac{1}{5} = \frac{2}{7} \xrightarrow{\text{+}} \frac{1}{5} = \frac{10 + 7}{35} = \frac{17}{35}$$

$$2. \frac{3}{5} - \frac{1}{3} = \frac{3}{5} \xrightarrow{\text{-}} \frac{1}{3} = \frac{9 - 5}{15} = \frac{4}{15}$$

- 20. Multiplying Fractions:** Multiplication of fractions does not require a common denominator. Just multiply numerators to create a new numerator, and multiply denominators to create a new denominator.

$$\text{e.g., } 1. \frac{3}{4} \cdot \frac{1}{2} = \frac{(3)(1)}{(4)(2)} = \frac{3}{8}$$

$$2. \frac{2}{3} \cdot \frac{2}{5} = \frac{(2)(2)}{(3)(5)} = \frac{4}{15}$$

- 21. Dividing Fractions:** To divide by a fraction, take the reciprocal of the divisor (the fraction doing the dividing) and then multiply the two terms.

$$\text{e.g., } 1. 2 \div \frac{1}{4} = 2 \cdot \frac{4}{1} = \frac{8}{1} = 8$$

$$2. \frac{\frac{2}{3}}{\frac{5}{6}} = \frac{2}{3} \cdot \frac{6}{5} = \frac{12}{15} = \frac{4}{5}$$

- 22. Converting Fractions to Decimals:** If the fraction already has a denominator that is 10, 100, 1,000, etc., the conversion is easy. The numerator of the fraction becomes the decimal. The placement of the decimal point is governed by the number of zeros in the denominator.

$$\text{e.g. Express } \frac{127}{1,000} \text{ in decimal form.}$$

In the numerator, count three places—one for each zero in 1,000—to the left from the right of the last digit: $\frac{127}{1,000} = 0.127$.

If there are fewer numbers in the numerator than there are decimal places, add zeros to the left of the number until there are enough decimal places.

$$\text{e.g., } \frac{3}{100} = 0.03$$

To convert a proper fraction with a denominator other than 10, 100, etc., first convert the fraction to the equivalent form using a denominator such as 10, 100, etc. To determine which denominator to use, divide the denominator of the fraction into 10, then into 100, then into 1,000, until a denominator that is evenly divisible by the denominator of the original fraction is found.

$$\text{e.g., } 1. \frac{2}{5} = \frac{(2)(2)}{(5)(2)} = \frac{4}{10} = 0.4$$

$$2. \frac{1}{4} = \frac{(1)(25)}{(4)(25)} = \frac{25}{100} = 0.25$$

$$3. \frac{3}{8} = \frac{(3)(125)}{(8)(125)} = \frac{375}{1,000} = 0.375$$

To convert proper fractions to decimals, dividing the denominator into the numerator is usually easier.

e.g., 1. $\frac{2}{5} = 5 \overline{)2.0} = 0.4$

2. $\frac{3}{8} = 8 \overline{)3.000} = 0.375$

To convert a mixed number into a decimal, convert the fractional part of the mixed number to a decimal as just discussed, and then place the whole number part of the mixed number to the left of the decimal point.

e.g., 1. $6\frac{1}{10} = 6.1$

(Convert $\frac{1}{10}$ to 0.1 and then place the 6 to the left of the decimal point.)

2. $3\frac{7}{8} = 3.875$

(Convert $\frac{7}{8}$ to 0.875 and then place the 3 to the left of the decimal point.)

To convert an improper fraction to a decimal, convert it to a mixed number and follow the procedure just outlined.

e.g., $\frac{9}{4} = 2\frac{1}{4} = 2.25$

- 23. Converting Decimals to Fractions:** The numerator of the fraction is the digit(s) to the right of the decimal point. The denominator is a 1 followed by the same number of zeros as the number of decimal places to the right of the decimal point.

e.g., $0.005 = \frac{5}{1,000} = \frac{1}{200}$

(0.005 has three decimal places, so the new denominator is 1 followed by 3 zeroes).

If a decimal has numbers to both the right and left of the decimal point, the conversion to a fraction results in a mixed number. The whole number part of the mixed number is the whole number part of the decimal.

e.g., 1. $1.75 = 1 + \frac{75}{100} = 1 + \frac{3}{4} = 1\frac{3}{4}$

2. $357.125 = 357 + \frac{125}{1,000} = 357 + \frac{1}{8} = 357\frac{1}{8}$

Memorize these decimal equivalents:

$$\frac{1}{2} = 0.50 \qquad \frac{1}{3} = 0.\overline{33} \qquad \frac{1}{4} = 0.25$$

$$\frac{1}{5} = 0.20 \qquad \frac{1}{6} = 0.1\overline{6} \qquad \frac{1}{7} = 0.\overline{142857}$$

$$\frac{1}{8} = 0.125 \qquad \frac{1}{9} = 0.\overline{11} \qquad \frac{1}{10} = 0.10$$

(Note: A bar over a digit or digits indicates that the digit or group of digits repeats.)

- 24. Adding and Subtracting Decimals:** To add or subtract decimals, line up the decimal points, fill in the appropriate number of zeros, and then add or subtract.

e.g., $0.25 + 0.1 + 0.825 = 0.25$

$$\begin{array}{r} 0.1 \\ + 0.825 \\ \hline = 0.250 \\ 0.100 \\ + 0.825 \\ \hline 1.175 \end{array}$$

- 25. Multiplying Decimals:** To multiply decimals, first multiply as with whole numbers and then adjust the decimal point. Count the total number of decimal places in the numbers being multiplied, count that many places to the left from the right of the final digit in the product, and put the decimal point there.

e.g., 1. $(0.1)(0.2)(0.3) = 0.006$

($1 \cdot 2 \cdot 3 = 6$, and there are three decimal places in the multiplication.)

2. $(0.10)(0.10)(0.10) = 0.001000 = 0.001$

($10 \cdot 10 \cdot 10 = 1,000$, and there are six decimal places in the multiplication.)

- 26. Dividing Decimals:** When the divisor (the quantity by which another quantity, the dividend, is to be divided) is a whole number, place a



decimal point in the quotient immediately above the decimal point in the dividend. Keep dividing until there is no remainder, adding zeros as needed to the right of the divisor.

$$\begin{array}{r} \text{e.g., } 2.5 \div 2 = 2 \overline{)2.50} \\ \underline{-2} \\ 05 \\ \underline{-4} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

When the divisor is a *decimal*, “clear” the fractional part of the decimal by moving the decimal points of both the dividend and divisor, the same number of spaces to the right.

$$\text{e.g., } 5 \div 2.5 = 50 \div 25 = 25 \overline{)50} \\ \underline{-50} \\ 0$$

- 27. Ratios:** A *ratio* is a statement about the relationship between two quantities. The ratio of two quantities, x and y , can be expressed as $x \div y$, x/y , or $x : y$.

$$\text{e.g., } 1. \frac{2}{5} = 2 : 5$$

$$2. \frac{\text{boys}}{\text{girls}} = \text{boys} : \text{girls} = \text{ratio of boys to girls}$$

$$3. \frac{\text{miles}}{\text{hour}} = \text{miles} : \text{hour} = \text{miles per hour}$$

- 28. Proportions:** A *proportion* is a statement of equality between two ratios.

$$\text{e.g., } \frac{3}{4} = \frac{9}{12}$$

With *direct variation*, ratios are directly related: the more of one quantity, the more of the other, and vice versa.

e.g., If 12 donuts cost \$3.60, how much do 4 donuts cost?

$$\begin{array}{l} \frac{\text{Total Cost for } X}{\text{Total Cost for } Y} = \frac{X}{Y} \\ \frac{\$3.60}{Y} = \frac{12}{4} \end{array}$$

$$\$3.60(4) = 12Y$$

$$Y = 3.60(4) \div 12 = \$1.20$$

With *inverse variation*, ratios are inversely related: an increase in one quantity is a decrease in the other. Use this method to solve inverse variation problems: first, set up an ordinary proportion, making sure that you group like quantities; then, take the reciprocal of the proportion’s right side; and finally, cross-multiply and solve for the unknown.

e.g., Traveling at a constant rate of 150 mph, a plane makes the trip from City P to City Q in four hours. How long will the trip take if the plane flies at a constant rate of 200 mph?

$$\frac{\text{Speed } X}{\text{Speed } Y} = \frac{\text{Time } X}{\text{Time } Y}$$

$$\frac{150 \text{ mph}}{200 \text{ mph}} = \frac{4 \text{ hours}}{Y \text{ hours}}$$

$$\frac{150}{200} = \frac{Y}{4}$$

$$Y = 4(150) \div 200 = 3 \text{ hours}$$

- 29. Percentage Conversions:** To *change any decimal to a percent*, move the decimal point two places to the right and add a percent sign. To *change a percent to a decimal*, reverse the process.

$$\text{e.g., } 1. 0.275 = 27.5\%$$

$$2. 0.03 = 3\%$$

$$3. 0.02\% = 0.0002$$

$$4. 120\% = 1.20$$

To *convert a fraction to a percent*, first convert the fraction to a decimal. Reverse the process for converting percents to fractions.

$$\text{e.g., } 1. \frac{3}{4} = 0.75 = 75\% = 0.75 = \frac{75}{100} = \frac{3}{4}$$

$$2. \frac{5}{8} = 0.625 = 62.5\% = \frac{625}{1,000} = \frac{5}{8}$$

30. Common Percent Problems: All percent problems have the same three components: “is,” “of,” and %. Depending on the form of the question, one of these three components is the unknown variable.

“What is $x\%$ of that?”

“This is what percent of that?”

“This is $x\%$ of what?”

Percent problems can be solved using several different methods. Two methods are outlined below.

Method 1: Write the statement as an equation, rewrite the percent as $\frac{\%}{100}$, and solve for the unknown.

e.g., 5 is 20% of what number?

$$5 = \frac{20}{100}x$$

$$x = \frac{(5)(100)}{20} = 25$$

Thus, 5 is 20% of 25.

Method 2: Since there are three parts to all percent problems (“is,” “of,” and %), use the following equation to solve for the unknown: $\frac{\text{is}}{\text{of}} = \frac{\%}{100}$.

e.g., 1. 20 is what percent of 50?

$$\% = x, \text{ “is”} = 20, \text{ “of”} = 50$$

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100}$$

$$\frac{20}{50} = \frac{x}{100}$$

$$x = \frac{(20)(100)}{50} = 40$$

2. What number is 20% of 25?

$$\% = 20, \text{ “is”} = x, \text{ “of”} = 25$$

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100}$$

$$\frac{x}{25} = \frac{20}{100}$$

$$x = \frac{(20)(25)}{100} = 5$$

Another common percent item is *change in amount*.

$$\text{Percent Change} = \frac{|\text{New Amount} - \text{Original Amount}|}{\text{Original Amount}}$$

The absolute value allows for percent decreases as well.

e.g., An item’s price is increased from \$3 to \$5. What is the percent increase in the price?

$$\frac{|\text{New Amount} - \text{Original Amount}|}{\text{Original Amount}} = \frac{5 - 3}{3}$$

$$= \frac{2}{3}$$

$$= 66\frac{2}{3}\%$$

31. Averages: To calculate an *average* (or mean), add together the quantities to be averaged; then divide that sum by the number of quantities added.

e.g., The average of 3, 7, and 8 is 6: $3 + 7 + 8 = 18$ and $18 \div 3 = 6$.

If solving for a *missing element* of an average, set up the average equation and solve for the unknown.

e.g., The average score on four tests is 90. If three scores are 89, 92, and 94, what is the fourth score?

$$\frac{89 + 92 + 94 + x}{4} = 90 \Rightarrow x = 85$$

In *weighted averages*, greater weight is given to one element than to another.



e.g., Four books cost \$6.00 each and two books cost \$3.00 each. What is the average cost of a book?

$$\frac{(4)(6) + (2)(3)}{4 + 2} = \frac{24 + 6}{6} = \frac{30}{6} = 5$$

- 32. Median:** The *median* is the middle value of a number set when arranged in ascending or descending order. The median of an even numbered set is the average of the two middle values, when the numbers are arranged in ascending or descending order.

e.g., The median of {8, 6, 34, 5, 17, 23} is:
 $\frac{8 + 17}{2} = 12.5$.

- 33. Mode:** The value that appears most frequently in a set of numbers is the *mode*.

e.g., The mode of {4, 5, 3, 4, 5, 1, 2, 3, 6, 4, 6} is 4.

- 34. Counting Principle:** To determine the number of ways that particular events can occur, multiply the number of ways that each event can occur.

e.g., 1. How many ways can you select one boy and one girl from a class of 15 girls and 13 boys?

$$(15)(13) = 195 \text{ ways}$$

2. In how many ways can 5 students sit in a row with 5 chairs?

$$(5)(4)(3)(2)(1) = 120 \text{ ways}$$

3. In how many ways can you fill three chairs given five students?

$$(5)(4)(3) = 60 \text{ ways}$$

- 35. Probability Principle:** The probability that an event will happen can be found from the fraction $\frac{\text{winning events}}{\text{total events}}$ or $\frac{\text{favorable events}}{\text{total events}}$.

e.g., From the set {−4, −3, −2, 0, 1, 6, 8, 1002}, a number is selected at random. What is the probability that the selected number is an even integer?

In the set, there are six even integers (−4, −2, 0, 6, 8, and 1002) of a total of eight

integers, so the probability is: $\frac{6}{8} = \frac{3}{4}$.

Algebra

1. Basic Operations:

Addition: $n + n = 2n$
 $n + m = n + m$

Subtraction: $3n - 2n = n$
 $n - m = n - m$

Multiplication: $n \cdot m = (n)(m) = nm$
 $(n)(0) = 0$

Division: $n \div m = \frac{n}{m}$
 $n \div 0 = \text{undefined}$

2. Powers: A *power* of a number indicates repeated multiplication.

e.g., 3 raised to the fifth power is equal to: $3^5 = (3)(3)(3)(3)(3) = 243$.

3. Exponents: An *exponent* is a number that indicates the operation of repeated multiplication. Exponents are notated as superscripts. The number being multiplied is the *base*.

e.g., 1. $2^3 = (2)(2)(2) = 8$

2. $5^4 = (5)(5)(5)(5) = 625$

Exponent Rules:

a) $x^m \cdot x^n = x^{m+n}$

b) $x^m \div x^n = x^{m-n}$

c) $(x^m)^n = x^{mn}$

d) $(xy)^m = x^m y^m$

e) $\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}$

f) $x^1 = x$, for any number x

g) $x^0 = 1$, for any number x , such that $x \neq 0$

h) 0^0 is undefined.

e.g., 1. $(2^3)(2^2) = (2 \cdot 2 \cdot 2)(2 \cdot 2) = 2^{3+2} = 2^5$

2. $(3^2)(3^3)(3^5) = 3^{2+3+5} = 3^{10}$

3. $2^4 \div 2^2 = \frac{(2)(2)(2)(2)}{(2)(2)} = 2^{4-2} = 2^2$

4. $(2^2)^3 = (2 \cdot 2)^3$
 $= (2 \cdot 2)(2 \cdot 2)(2 \cdot 2)$
 $= 2^{(2)(3)}$
 $= 2^6$

5. $(2 \cdot 3)^2 = (2 \cdot 3)(2 \cdot 3)$
 $= (2 \cdot 2)(3 \cdot 3)$
 $= 2^2 \cdot 3^2$
 $= 4 \cdot 9$
 $= 36$

6. $(2^3 \cdot 3^2)^2 = 2^{3 \cdot 2} \cdot 3^{2 \cdot 2} = 2^6 \cdot 3^4$

7. $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$

8. $\left(\frac{3^3 \cdot 5^5}{3^2 \cdot 5^2}\right)^2 = (3^{3-2} \cdot 5^{5-2})^2$
 $= (3^1 \cdot 5^3)^2$
 $= (3^2)(5^6)$

A negative exponent signifies a fraction, indicating the *reciprocal* of the base.

e.g., 1. $x^{-1} = \frac{1}{x}$

2. $2x^{-1} = 2\left(\frac{1}{x}\right) = \frac{2}{x}$

3. $4^{-2} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$



- 4. Roots:** The *root* of a number is a number that is multiplied a specified number of times to give the original number. Square root = $m^{\frac{1}{2}} = \sqrt{m}$. Cube root = $m^{\frac{1}{3}} = \sqrt[3]{m}$.

e.g., 1. $\sqrt{4} = 4^{\frac{1}{2}} = 2$

2. $\sqrt[3]{8} = 8^{\frac{1}{3}} = 2$

3. $\sqrt{125} = 125^{\frac{1}{2}}$
 $= (25 \cdot 5)^{\frac{1}{2}}$
 $= (25)^{\frac{1}{2}} (5)^{\frac{1}{2}}$
 $= (\sqrt{25})(\sqrt{5})$
 $= 5\sqrt{5}$

4. $\sqrt{\frac{4}{9}} = \left(\frac{4}{9}\right)^{\frac{1}{2}} = \frac{4^{\frac{1}{2}}}{9^{\frac{1}{2}}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

- 5. Basic Algebraic Operations:** Algebraic operations are the same as for arithmetic, with the addition of unknown quantities. Manipulate operations in the same way, combining (adding and subtracting) only like terms. Like terms have the same variables with the same exponents.

e.g., 1. $x^2 - 3x + 5x - 3x^2 = -2x^2 + 2x$

2. $(x^2)(x^3) = x^{2+3} = x^5$

3. $4x^3y^4 \div 2xy^3 = 2x^2y$

4. $\frac{5}{x} + \frac{3}{x} = \frac{5+3}{x} = \frac{8}{x}$

5. $\left(\frac{x^2y^3}{z}\right)\left(\frac{x^3y^2}{wz}\right) = \frac{x^5y^5}{wz^2}$

- 6. Multiplying Polynomials:** A *polynomial* is an algebraic expression with more than one term. A *binomial* is a polynomial consisting of exactly

two terms. When multiplying two binomials, use the *FOIL* (*F*irst, *O*uter, *I*nner, *L*ast) *method*:

$$(x + y)(x + y) = ?$$

Multiply the *first* terms: $x \cdot x = x^2$

Multiply the *outer* terms: $x \cdot y = xy$

Multiply the *inner* terms: $y \cdot x = yx = xy$

Multiply the *last* terms: $y \cdot y = y^2$

Combine like terms: $(x + y)(x + y) = x^2 + 2xy + y^2$

e.g., $(x - y)(x - y) = ?$

First: $(x)(x) = x^2$

Outer: $(x)(-y) = -xy$

Inner: $(-y)(x) = -xy$

Last: $(-y)(-y) = y^2$

Combine: $x^2 - xy - xy + y^2 = x^2 - 2xy + y^2$

If the two polynomials are not binomials, do the following:

e.g., $(x + y)(x^2 + 2xy + y^2)$
 $= x(x^2) + x(2xy) + x(y^2) + y(x^2) +$
 $y(2xy) + y(y^2)$
 $= x^3 + 2x^2y + xy^2 + x^2y + 2xy^2 + y^3$
 $= x^3 + 3x^2y + 3xy^2 + y^3$

Memorize these common patterns:

$$(x + y)^2 = (x + y)(x + y) = x^2 + 2xy + y^2$$

$$(x - y)^2 = (x - y)(x - y) = x^2 - 2xy + y^2$$

$$(x + y)(x - y) = x^2 - y^2$$

- 7. Factoring:** *Factoring* is the reverse of multiplication. There are three common factoring situations.

- a) If all of the terms in an expression contain a common factor, then it can be factored out of each term. Do this first, if possible.

e.g., 1. $ab + ac + ad = a(b + c + d)$

2. $x^2 + x^3 + x^4 = x^2(1 + x + x^2)$

$$3. 3xy + xz + 4x = x(3y + z + 4)$$

- b) Algebraic expressions are often one of three common patterns.

e.g., 1. $x^2 + 2xy + y^2 = (x + y)(x + y)$
 $= (x + y)^2$

2. $x^2 - 2xy + y^2 = (x - y)(x - y)$
 $= (x - y)^2$

3. $x^2 - y^2 = (x - y)(x + y)$

- c) Occasionally, expressions do not fall into one of the two categories above. To factor the expression, which is usually in the form $ax^2 + bx + c$, set up the following blank diagram: $(\quad)(\quad)$. Fill in the diagram by answering the following questions:

- What factors produce the first term, ax^2 ?
- What factors produce the last term, c ?
- Which of the possible factors, when added together, produce the middle term, bx ?

e.g., 1. $x^2 + 3x + 2 = (x + 2)(x + 1)$

2. $x^2 + 4x - 12 = (x + 6)(x - 2)$

- 8. Solving Linear Equations:** An equation that contains variables only of the first power is a linear equation. You can add, subtract, multiply, and divide both sides of an equation by the same value without changing the statement of equality. (You cannot multiply or divide by zero.) To find the value of a variable, isolate the variable on one side of the equation and solve.

e.g., 1. $4x + 2 = 2x + 10$
 $4x + 2 - 2x = 2x + 10 - 2x$
 $2x + 2 = 10$
 $2x + 2 - 2 = 10 - 2$
 $2x = 8$
 $\frac{2x}{2} = \frac{8}{2}$
 $x = 4$

2. $\frac{2x + 6}{2} = 9$
 $\left(\frac{2x + 6}{2}\right)(2) = 9(2)$
 $2x + 6 = 18$
 $2x + 6 - 6 = 18 - 6$
 $2x = 12$
 $\frac{2x}{2} = \frac{12}{2}$
 $x = 6$

- 9. Solving Quadratic Equations:** Equations that involve variables of the second power are called quadratic equations and may have zero, one, or two real solutions.

- a) If possible, take the square root of both sides.

e.g., $x^2 = 25$
 $x = \pm 5$

- b) Otherwise, arrange all terms on the left side of equation so that the right side of equation is zero: $ax^2 + bx + c = 0$. Factor the left side of the equation and set each binomial equal to zero. Solve for the unknown.

e.g., 1. If $x^2 - 2x = 3$, what is x ?

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = \{-1, 3\}$$

2. If $x^2 - 3x = 4$, what is x ?

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x = \{-1, 4\}$$

- c) The quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, may also be used to solve quadratic equations.



e.g., If $3 - x = 2x^2$, what is x ?

$$\begin{aligned}
 2x^2 + x - 3 &= 0; a = 2; b = 1; c = -3 \\
 x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
 &= \frac{-1 \pm \sqrt{1^2 - 4(2)(-3)}}{2(2)} \\
 &= \frac{-1 \pm \sqrt{1 + 24}}{4} \\
 &= \frac{-1 \pm 5}{4} \\
 &= \left\{ -\frac{3}{2}, 1 \right\}
 \end{aligned}$$

- 10. Solving Simultaneous Equations:** Given two equations with two variables, the equations may be solved simultaneously for the values of the two variables. There are several methods for solving simultaneous equations.

Method 1—Substitution: Solve one equation for one variable and substitute this into the other equation to find the other variable. Plug back into the first equation.

e.g., If $2x - y = 6$ and $3x + 2y = 16$, what is x and y ?

$$\begin{aligned}
 \text{Solve for } y: \quad 2x - y &= 6 \\
 -y &= 6 - 2x \Rightarrow y = 2x - 6
 \end{aligned}$$

$$\begin{aligned}
 \text{Substitute:} \quad 3x + 2y &= 16 \\
 3x + 2(2x - 6) &= 16 \\
 3x + 4x - 12 &= 16 \\
 7x &= 28 \\
 x &= 4
 \end{aligned}$$

$$\text{Substitute:} \quad y = 2x - 6 = 2(4) - 6 = 2$$

Method 2—Elimination: Make the coefficients of one variable equal and then add (or subtract) the two equations to eliminate one variable.

e.g., If $2x - y = 6$ and $3x + 2y = 16$, what is x and y ?

$$\begin{aligned}
 \text{Combine:} \quad 2[2x - y = 6] \\
 + 3x + 2y = 16 \\
 \hline
 7x = 28 \\
 x = 4
 \end{aligned}$$

$$\begin{aligned}
 \text{Substitute:} \quad 2x - y &= 6 \\
 2(4) - y &= 6 \\
 y &= 2
 \end{aligned}$$

- 11. Inequalities:** The fundamental rule for working with inequalities is similar to that for working with equalities. The same value may be added or subtracted to each side of an inequality without changing the inequality. Each side may be multiplied or divided by the same *positive* value without changing the direction of the inequality.

$$\begin{aligned}
 \text{e.g., 1. } 5 &> 2 \\
 5 + 25 &> 2 + 25 \\
 30 &> 27
 \end{aligned}$$

$$\begin{aligned}
 \text{2. } 24 &> 20 \\
 24(2) &> 20(2) \\
 48 &> 40
 \end{aligned}$$

$$\begin{aligned}
 \text{3. } 24 &> 20 \\
 24 \div 4 &> 20 \div 4 \\
 6 &> 5
 \end{aligned}$$

To multiply or divide by a *negative* number, reverse the direction of the inequality.

$$\begin{aligned}
 \text{e.g., 1. } 4 &> 2 \\
 4(-2) &< 2(-2) \\
 -8 &< -4
 \end{aligned}$$

$$\begin{aligned}
 \text{2. } 4 &> 2 \\
 4 \div (-2) &< 2 \div (-2) \\
 -2 &< -1
 \end{aligned}$$

- 12. Slope:** The *slope*, m , of a line describes its steepness. It is defined as the change in y -values divided by the change in x -values, or rise over run.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{rise}}{\text{run}}$$

e.g., The slope of the line containing points $(-3,5)$ and $(2,7)$ is equal to: $m = \frac{y_2 - y_1}{x_2 - x_1} =$

$$\frac{7 - 5}{2 - (-3)} = \frac{2}{5}.$$

13. Linear Equations:

Slope-Intercept Form: $y = mx + b;$

$$\text{Slope: } m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-Slope Form: $y - y_1 = m(x - x_1)$

Standard Form: $ax + by = c, m = -\frac{a}{b}$

14. Distance Formula:

The distance between two points can be found using the *distance formula*:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where (x_1, y_1) and (x_2, y_2) are the given points.

e.g., What is the distance between points $(-1,4)$ and $(7,3)$?

$$d = \sqrt{[7 - (-1)]^2 + (3 - 4)^2} = \sqrt{64 + 1} = \sqrt{65}$$

15. Midpoint Formula:

The midpoint between two points, (x_1, y_1) and (x_2, y_2) , is found using the *midpoint formula*:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

e.g., The midpoint between points $(-3,6)$ and

$$(4,-9) \text{ is equal to: } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-3 + 4}{2}, \frac{6 + (-9)}{2} \right) = \left(\frac{1}{2}, -\frac{3}{2} \right).$$

16. Functions:

A function is a set of ordered pairs (x, y) such that for each value of x , there is exactly one value of y . The set of x -values for which the set is defined is called the domain of the function. The set of corresponding values of y is called the range of the function.

e.g., What is the domain and range for the function $f(x) = x^2$?

f represents the function and x represents values in the domain of the function. $f(x)$ represents values in the range of the function. Since x can be any real number, the domain is the set of all real numbers. We square the value of x to obtain $f(x)$. Squaring any real number yields a number of zero or more. Thus, the range is the set of all non-negative numbers.



Common Equations

- 1. Distance:** Distance = (Rate)(Time). Given two of the three values, any unknown may be solved for by rearranging the equation.

e.g., After driving constantly for four hours, Olivia reached her destination: 200 miles from where she started. What was her average rate of travel?

$$\begin{aligned} \text{Distance} &= (\text{Rate})(\text{Time}) \\ \text{Rate} &= \frac{\text{Distance}}{\text{Time}} = \frac{200 \text{ miles}}{4 \text{ hours}} = 50 \text{ mph} \end{aligned}$$

- 2. Simple Interest:** $I_{\text{simple}} = Prt$, where P is the principal, r is the rate, and t is the time period.

e.g., With a principal of \$1,200 and a rate of 10 percent per year, what was the interest earned over one month?

$$\begin{aligned} I_{\text{simple}} &= Prt \\ &= (\$1,200) \left(\frac{0.10}{\text{year}} \right) \left(\frac{1 \text{ year}}{12 \text{ months}} \right) = \$10 \end{aligned}$$

- 3. Compound Interest:** $I_{\text{compound}} = P \left(1 + \frac{r}{C} \right)^{tC} - P$,

where P is the principal, r is the rate, and t is the time period, and C is the number of times the interest is compounded annually.

e.g., A principal of \$1,000, compounded with interest of 15% per year, earns how much compound interest over 5 years?

$$\begin{aligned} I_{\text{compound}} &= P \left(1 + \frac{r}{C} \right)^{tC} - P \\ &= \$1,000 \left(1 + \frac{0.15}{1} \right)^{5 \cdot 1} - \$1,000 \\ &\approx \$2,011 - \$1,000 = \$1,011 \end{aligned}$$

- 4. Combined Work Rates:** $\text{Rate}_1 + \text{Rate}_2 = \text{Rate}_3$

e.g., Machine A washes four loads in 60 minutes and Machine B washes one load in 30 minutes. How many loads will both machines together wash in 20 minutes?

$$\begin{aligned} \frac{x \text{ loads}}{20 \text{ min.}} &= \frac{4 \text{ loads}}{60 \text{ min.}} + \frac{1 \text{ load}}{30 \text{ min.}} \\ &= \frac{4 \text{ loads}}{60 \text{ min.}} + \frac{2 \text{ loads}}{60 \text{ min.}} \\ x &= 20 \left(\frac{4}{60} + \frac{2}{60} \right) \\ &= \frac{6(20)}{60} \\ &= 2 \text{ loads} \end{aligned}$$

- 5. Mixed Denominations:** When an item gives information that involves *mixed denominations* (e.g., different prices for same item, tickets, colors, etc.), set up simultaneous equations and solve the system of equations for the desired unknown quantity.

e.g., The store sold apples for \$0.20 and oranges for \$0.50 each. A total of 50 apples and oranges were bought for \$19. How many apples and how many oranges were bought?

If x equals the number of apples and y the number of oranges, $x = 50 - y$. Therefore:

$$\begin{aligned} (0.20)x + (0.50)y &= 19 \\ 0.2(50 - y) + 0.5y &= 19 \\ 10 - 0.2y + 0.5y &= 19 \\ 0.3y &= 9 \\ y &= 30 \\ x = 50 - y & \\ &= 50 - 30 \\ &= 20 \end{aligned}$$

- 6. Mixture of Concentrations or Values:** A *mixture item* is one in which two quantities of different items with different concentrations or values are mixed together and a new quantity (the sum of the two) and concentration or value are created.

$$Q_1C_1 + Q_2C_2 = (Q_1 + Q_2)C_3$$

e.g., How many liters of a juice that is 10% orange juice must be added to three liters of another juice that is 15% orange juice to produce a mixture that is 12% orange juice?

$$\begin{aligned} Q_1C_1 + Q_2C_2 &= (Q_1 + Q_2)C_3 \\ Q(0.10) + (3)(0.15) &= (Q + 3)(0.12) \\ 0.1Q + 0.45 &= 0.12Q + 0.36 \\ 0.45 - 0.36 &= 0.12Q - 0.1Q \\ 0.09 &= 0.02Q \\ Q &= 4.5 \end{aligned}$$

7. Markup, Cost, and Revenue: $R = (1 + M)C$, where R is revenue, M is markup, and C is cost.

e.g., The revenue from an item marked up 25% is \$120. What was the original cost?

$$C = \frac{R}{1 + M} = \frac{120}{1 + 0.25} = \$96$$

Geometry

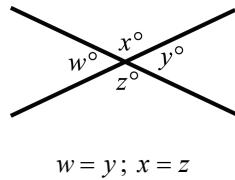
1. Lines and Angles:

a) Symbols:

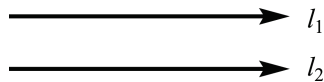
- \overline{AB} : line segment with endpoints A and B
- \overrightarrow{AB} : infinite ray from A through B
- \overleftrightarrow{AB} : infinite line through both A and B
- $l_1 \parallel l_2$: parallel lines
- \perp : perpendicular
- \square : right angle

b) Facts about Lines and Angles:

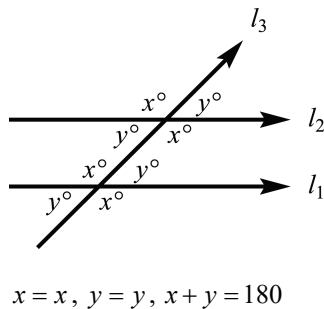
Vertical angles are equal:



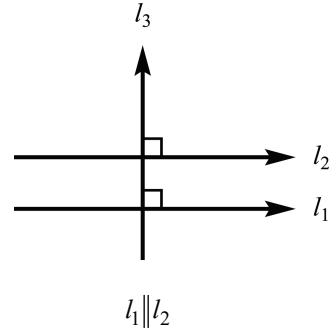
Two extended lines that do not intersect regardless of length are *parallel* to each other:



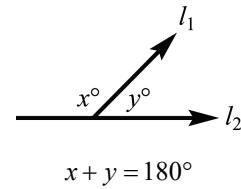
Parallel lines intersected by a third line, the transversal, create the following angles:



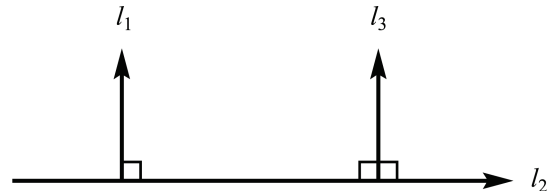
Two lines *perpendicular* to the same line are parallel to each other:



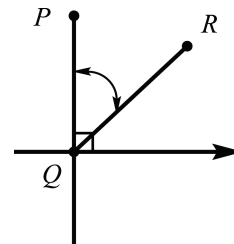
There are 180° in a *straight line*:



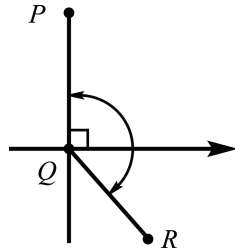
There are 90° in a *right angle* and two right angles form a straight line:



An angle less than 90° is an *acute angle*. In the following figure, $\angle PQR$ is an acute angle:

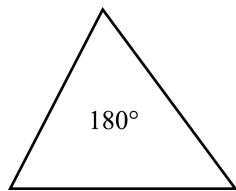


An angle greater than 90° is an *obtuse angle*. In the following figure, $\angle PQR$ is an obtuse angle:



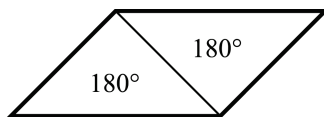
2. **Polygons:** A *polygon* is a closed figure created by three or more lines. The sum of the interior angles of any polygon is $180(n-2)$, where n = the number of sides of the polygon. The sum of the measures of the exterior angles of a polygon is 360° for all polygons.

A *triangle* is any polygon with exactly three sides.



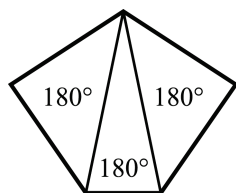
$$180(n-2) = 180(3-2) = 180$$

A *quadrilateral* is any polygon with exactly four sides. Opposite sides of a *parallelogram* are equal and parallel.



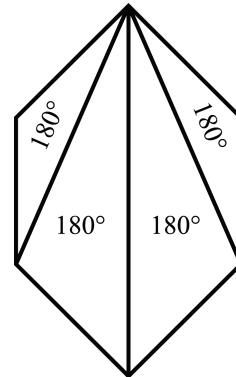
$$180(n-2) = 180(4-2) = 360$$

A *pentagon* is any polygon with exactly five sides.



$$180(n-2) = 180(5-2) = 540$$

A *hexagon* is any polygon with exactly six sides.

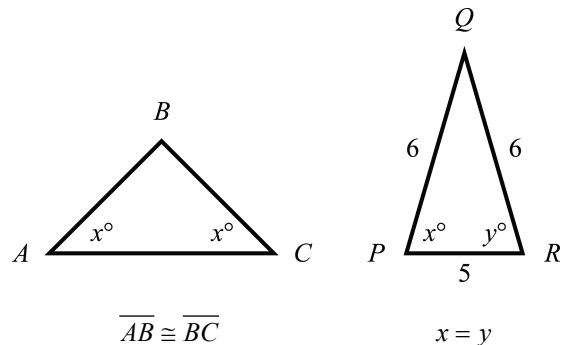


$$180(n-2) = 180(6-2) = 720$$

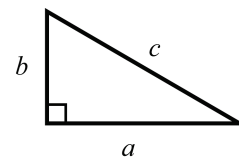
3. **Triangles:** A *triangle* is a 3-sided figure. Within a given triangle, the larger the angle, the longer the opposite side; conversely, the longer the side, the larger the opposite angle.

A triangle with two equal sides is an *isosceles* triangle. A triangle with three equal sides is an *equilateral* triangle.

Within a given triangle, if two sides are equal, their opposite angles are equal, and vice versa:

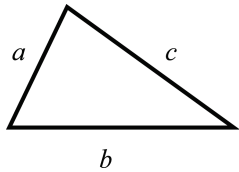


The sides of every right triangle follow the *Pythagorean theorem*: the square of the hypotenuse is equal to the sum of the squares of the other two sides.



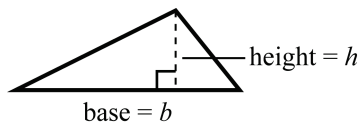
$$c^2 = a^2 + b^2$$

The *perimeter of a triangle* is the sum of the lengths of the three sides:



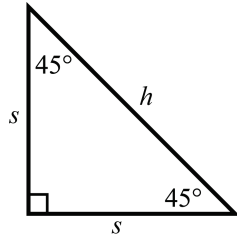
$$\text{Perimeter}_{\text{triangle}} = a + b + c$$

The *area of a triangle* is one-half times the base times the height:



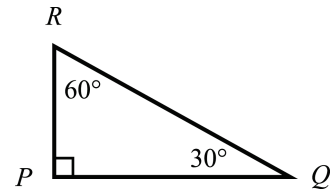
$$\text{Area}_{\text{triangle}} = \frac{1}{2}(bh) = \frac{bh}{2}$$

In a $45^\circ\text{-}45^\circ\text{-}90^\circ$ triangle, the length of the hypotenuse is equal to the length of either leg multiplied by the square root of two:



$$h = (s)(\sqrt{2})$$

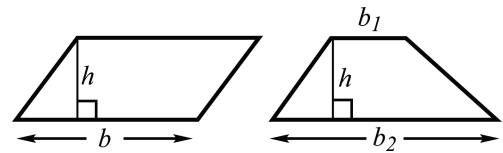
In a $30^\circ\text{-}60^\circ\text{-}90^\circ$ triangle, the length of the side opposite the 30° angle is equal to one-half the length of the hypotenuse and the length of the side opposite the 60° angle is equal to one-half the length of the hypotenuse multiplied by $\sqrt{3}$:



$$\overline{PR} = \frac{\overline{QR}}{2}$$

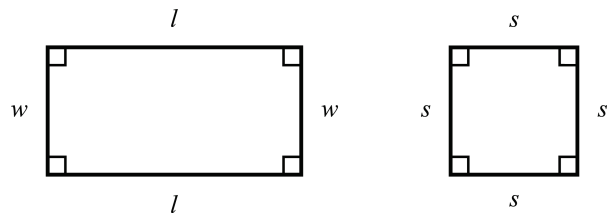
$$\overline{PQ} = \frac{\sqrt{3}(\overline{QR})}{2}$$

4. Parallelograms and Trapezoids: Again, a *parallelogram* is a quadrilateral in which both pairs of opposite sides are parallel. A *trapezoid* is a quadrilateral with only two parallel sides.



$$\text{Area}_{\text{parallelogram}} = bh \quad \text{Area}_{\text{trapezoid}} = \frac{(b_1 + b_2)h}{2}$$

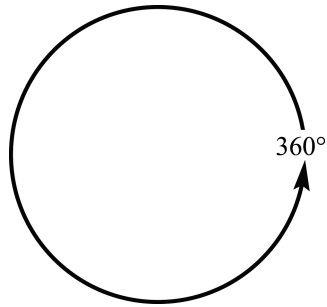
5. Rectangles and Squares: A *rectangle* is any four-sided figure that has four right angles. A *square* is a rectangle with four equal sides:



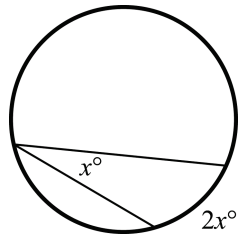
$$\text{Perimeter}_{\text{rectangle}} = 2(w+l) \quad \text{Perimeter}_{\text{square}} = 4s$$

$$\text{Area}_{\text{rectangle}} = lw \quad \text{Area}_{\text{square}} = s^2$$

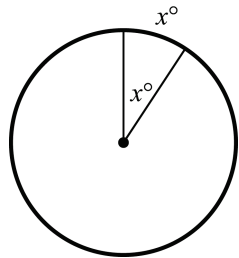
6. Circles: The distance from the center of a circle to any point on the circle is the *radius*. A line segment that passes through the center of a circle and that has endpoints on the circle is called the *diameter*. The diameter of a circle is twice the radius. There are 360° of *arc* in a circle:



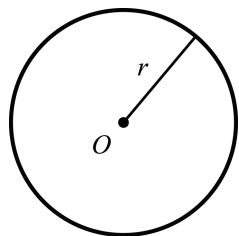
An angle inscribed in a circle intercepts an arc that is twice its measure:



An angle whose vertex is at the center of a circle intercepts an arc of the same measure:



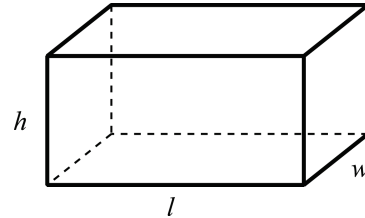
The *circumference of a circle* is the radius times 2π . The *area of a circle* is the radius squared times π .



$$\text{Circumference}_{\text{circle}} = 2\pi r$$

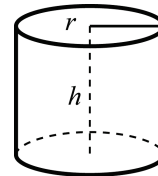
$$\text{Area}_{\text{circle}} = \pi r^2$$

7. Solid Geometry: Solid geometry refers to three-dimensional figures. *Volume* is a three-dimensional quantity. The *volume of a rectangular solid* is the length times the width times the height:



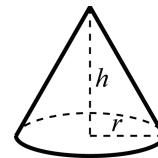
$$\text{Volume}_{\text{solid}} = lwh$$

The *volume of a cylinder* is the height times the radius squared times π .



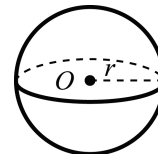
$$\text{Volume}_{\text{cylinder}} = h(\pi r^2)$$

The *volume of a right circular cone* is one-third of the height times the radius squared times π .



$$\text{Volume}_{\text{right circular cone}} = \frac{h(\pi r^2)}{3}$$

The *surface area of a sphere* is four times π times the radius squared. The *volume of a sphere* is four-thirds times π times the radius cubed.

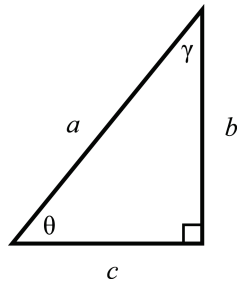


$$\text{Area}_{\text{surface of sphere}} = 4\pi r^2$$

$$\text{Volume}_{\text{sphere}} = \frac{4\pi r^3}{3}$$

Trigonometry

1. Basic Trigonometric Functions: The three basic trigonometric functions are sine, cosine, and tangent. The functions can be viewed in relationship to a right triangle.



The *hypotenuse* is the side opposite the right angle. The *side opposite* is the side opposite the referenced angle. The *side adjacent* is the side next to the referenced angle, but not the hypotenuse.

Memorize these trigonometric definitions:

sine:
$$\sin \theta = \frac{\text{side opposite } \angle \theta}{\text{hypotenuse}} = \frac{b}{a}$$

cosine:
$$\cos \theta = \frac{\text{side adjacent } \angle \theta}{\text{hypotenuse}} = \frac{c}{a}$$

tangent:
$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{side opposite } \angle \theta}{\text{side adjacent } \angle \theta} = \frac{b}{c}$$

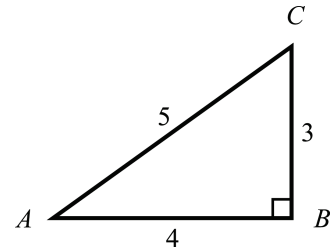
cosecant:
$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hypotenuse}}{\text{side opposite } \angle \theta} = \frac{a}{b}$$

secant:
$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hypotenuse}}{\text{side adjacent } \angle \theta} = \frac{a}{c}$$

cotangent:
$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{side adjacent } \angle \theta}{\text{side opposite } \angle \theta} = \frac{c}{b}$$

SOH-CAH-TOA can be used to memorize the sine, cosine, and tangent ratios.

e.g., Find the values of $\sin A$, $\cos A$, $\tan A$, $\csc A$, $\sec A$, $\cot A$, $\sin C$, $\cos C$, $\tan C$, $\csc C$, $\sec C$, and $\cot C$ in the triangle below.



$$\sin A = \frac{\text{side opp. } \angle A}{\text{hyp.}} = \frac{3}{5}$$

$$\sin C = \frac{\text{side opp. } \angle C}{\text{hyp.}} = \frac{4}{5}$$

$$\cos A = \frac{\text{side adj. } \angle A}{\text{hyp.}} = \frac{4}{5}$$

$$\cos C = \frac{\text{side adj. } \angle C}{\text{hyp.}} = \frac{3}{5}$$

$$\tan A = \frac{\text{side opp. } \angle A}{\text{side adj. } \angle A} = \frac{3}{4}$$

$$\tan C = \frac{\text{side opp. } \angle C}{\text{side adj. } \angle C} = \frac{4}{3}$$

$$\csc A = \frac{\text{hyp.}}{\text{side opp. } \angle A} = \frac{5}{3}$$

$$\csc C = \frac{\text{hyp.}}{\text{side opp. } \angle C} = \frac{5}{4}$$

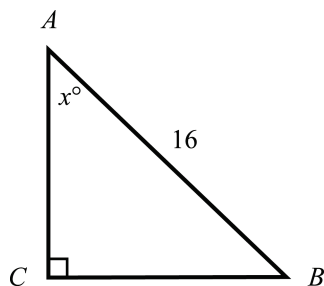
$$\sec A = \frac{\text{hyp.}}{\text{side adj. } \angle A} = \frac{5}{4}$$

$$\sec C = \frac{\text{hyp.}}{\text{side adj. } \angle C} = \frac{5}{3}$$

$$\cot A = \frac{\text{side adj. } \angle A}{\text{side opp. } \angle A} = \frac{4}{3}$$

$$\cot C = \frac{\text{side adj. } \angle C}{\text{side opp. } \angle C} = \frac{4}{3}$$

e.g., In the right triangle below, $\sin x = 0.75$. What is the length of \overline{AC} ?



$$\sin x = \frac{\text{side opposite } \angle x}{\text{hypotenuse}}$$

$$\begin{aligned} \frac{3}{4} &= \frac{\overline{BC}}{16} \\ \overline{BC} &= \frac{3(16)}{4} \\ &= 12 \end{aligned}$$

Therefore, using the Pythagorean theorem:

$$\begin{aligned} (\overline{AB})^2 &= (\overline{AC})^2 + (\overline{BC})^2 \\ 16^2 &= (\overline{AC})^2 + 12^2 \\ (\overline{AC})^2 &= 256 - 144 \\ &= 112 \\ \overline{AC} &= \sqrt{112} \\ &= \sqrt{16(7)} \\ &= 4\sqrt{7} \end{aligned}$$

2. Useful Trigonometric Relationships:

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{1}{\cot x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

Note: $\sin^2 x$ is the equivalent of $(\sin x)^2$.

e.g., If $\sin x = \frac{3}{5}$ and $\cos x = \frac{4}{5}$, what is the value of $\tan x$, $\csc x$, $\sec x$, and $\cot x$?

$$\tan x = \frac{\sin x}{\cos x} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\csc x = \frac{1}{\sin x} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

$$\sec x = \frac{1}{\cos x} = \frac{1}{\frac{4}{5}} = \frac{5}{4}$$

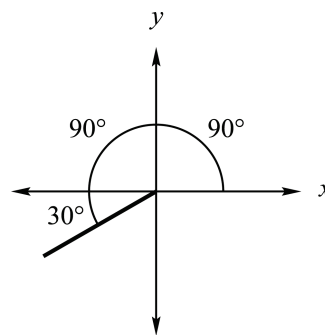
$$\cot x = \frac{1}{\tan x} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$$

e.g., If $\sin^2 x = \frac{1}{4}$, what is the value of $\cos^2 x$?

$$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{4} = \frac{3}{4}$$

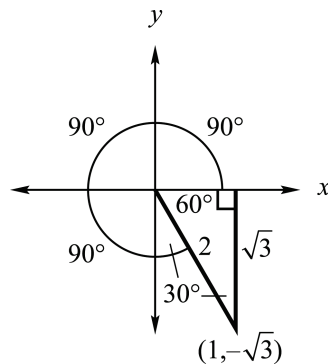
3. Graphing Using Trigonometry: Frequently, trigonometry is used in graphing. Angles on a graph are customarily measured in a counterclockwise manner from the positive x -axis.

e.g., 1. In the figure below, the angle shown has a measure of 210° : $90^\circ + 90^\circ + 30^\circ = 210^\circ$.





2. Without using a calculator, determine the values of $\cos 300^\circ$ and $\sin 300^\circ$.



Notice that after drawing the angle, we create a right triangle by drawing a perpendicular from the end of the angle to the x -axis. The hypotenuse of any such triangle is always considered to be positive. However, the point at the end of the original angle may not always have positive coordinates. You may make up any length desired for one side of the triangle. Then, use geometry or trigonometry with the angles inside the triangle to get the remaining lengths. In this example, we let the hypotenuse equal 2 and used geometry or trigonometry to get 1 and $\sqrt{3}$ as shown on the figure. Notice that the point at the end of our original 300° angle has a positive x -coordinate, 1, and a negative y -coordinate, $-\sqrt{3}$. We must use those signed values. The 300° angle leaves us 60° from the closer x -axis. Thus, $\cos 300^\circ$ uses the 60° angle:

$$\cos 300^\circ = \frac{\text{side adjacent } \angle 300^\circ}{\text{hypotenuse}} = \frac{1}{2}$$

$$\sin 300^\circ = \frac{\text{side opposite } \angle 300^\circ}{\text{hypotenuse}} = -\frac{\sqrt{3}}{2}$$

Formula Summary

ARITHMETIC, ALGEBRA, AND COORDINATE GEOMETRY

$$\text{Average} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n}; x_1, x_2, x_3, \dots, x_n = \text{values}; n = \text{total numbers}$$

$$\text{Finding Percent: } \frac{\text{is}}{\text{of}} = \frac{\%}{100}$$

$$\text{Percent Change} = \frac{|\text{New Amount} - \text{Original Amount}|}{\text{Original Amount}}$$

$$\text{Distance} = \text{Rate} \cdot \text{Time}$$

$$\text{Combined Work Rate: } \text{Rate}_3 = \text{Rate}_1 + \text{Rate}_2$$

$$\text{Mixture Problems: } (Q_1 + Q_2)C_3 = Q_1C_1 + Q_2C_2; Q = \text{quantities}; C = \text{concentrations}$$

$$\text{Interest}_{\text{simple}} = \text{Principal} \cdot \text{Rate} \cdot \text{Time}$$

$$\text{Interest}_{\text{compound}} = (\text{Principal}) \left(1 + \frac{\text{Rate}}{\# \text{ of Times Compounded Annually}} \right)^{(\text{Time Period})(\# \text{ of Times Compounded Annually})} - \text{Principal}$$

$$\text{Revenue} = (1 + \text{Markup})(\text{Cost})$$

$$\text{Slope-Intercept Linear Equation: } y = mx + b$$

$$\text{Slope of a Line: } m = \frac{y_2 - y_1}{x_2 - x_1}$$

**GEOMETRY**

$$\text{Perimeter}_{\text{square}} = 4s; s = \text{side}$$

$$\text{Perimeter}_{\text{rectangle}} = 2l + 2w; l = \text{length}; w = \text{width}$$

$$\text{Perimeter}_{\text{triangle}} = a + b + c; a, b, \text{ and } c \text{ are the sides}$$

$$\text{Circumference}_{\text{circle}} = 2\pi r; \pi \approx 3.14; r = \text{radius}$$

$$\text{Area}_{\text{square}} = s^2; s = \text{length of side}$$

$$\text{Area}_{\text{rectangle}} = lw; l = \text{length}; w = \text{width}$$

$$\text{Area}_{\text{parallelogram}} = bh; b = \text{base}; h = \text{height}$$

$$\text{Area}_{\text{trapezoid}} = \frac{(b_1 + b_2)h}{2}; b = \text{base}; h = \text{height}$$

$$\text{Area}_{\text{triangle}} = \frac{bh}{2}; b = \text{base}; h = \text{height}$$

$$\text{Area}_{\text{circle}} = \pi r^2; \pi \approx 3.14; r = \text{radius}$$

SOLID GEOMETRY

$$\text{Volume}_{\text{cube}} = s^3; s = \text{side}$$

$$\text{Volume}_{\text{rectangular solid}} = lwh; l = \text{length}; w = \text{width}; h = \text{height}$$

$$\text{Volume}_{\text{cylinder}} = \pi r^2 h; \pi \approx 3.14; r = \text{radius}; h = \text{height}$$

$$\text{Volume}_{\text{cone}} = \frac{\pi r^2 h}{3}; \pi \approx 3.14; r = \text{radius}; h = \text{height}$$

$$\text{Area}_{\text{surface of sphere}} = 4\pi r^2; \pi \approx 3.14; r = \text{radius}$$

$$\text{Volume}_{\text{sphere}} = \frac{4\pi r^3}{3}; \pi \approx 3.14; r = \text{radius}$$

POLYGONS

45°-45°-90° Triangle: $h = s\sqrt{2}$; h = hypotenuse; s = length of either leg

30°-60°-90° Triangle: $a = \frac{h}{2}$; a = side opposite $\angle 30^\circ$; h = hypotenuse

30°-60°-90° Triangle: $b = \frac{\sqrt{3}(h)}{2}$; b = side opposite $\angle 60^\circ$; h = hypotenuse

Pythagorean Theorem (Right Triangles): $c^2 = a^2 + b^2$; c = hypotenuse; a and b = legs

Sum of Interior Angles of Polygon: $S = 180(n - 2)$; n = number of sides of the polygon

TRIGONOMETRY

$$\sin \theta = \frac{\text{side opposite } \angle \theta}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{side adjacent } \angle \theta}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{side opposite } \angle \theta}{\text{side adjacent } \angle \theta}$$

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hypotenuse}}{\text{side opposite } \angle \theta}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hypotenuse}}{\text{side adjacent } \angle \theta}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{side adjacent } \angle \theta}{\text{side opposite } \angle \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$



Glossary of Terms

Absolute Value—the value of a number when the sign is not considered; the distance on the number line between the number and 0

Acute Angle—an angle with a measure of less than 90°

Adjacent Angles—angles that share a common side and a common vertex

Area—the space within a closed plane figure, measured in square units

Binomial—an algebraic expression with two terms

Circle—the set of all points in a plane that are equidistant from a center point

Circumference—the distance around a circle

Coefficient—the number in front of a term

Complementary Angles—two angles with a sum of 90°

Composite Numbers—a number that can be divided evenly by more than itself and one

Cube (1)—a six-sided solid with all six faces being equal-sized squares

Cube (2)—the result when a number is multiplied by itself twice

Cube Root—a number that when raised to the third power will yield a second given number

Denominator—the bottom term of a fraction

Diameter—a line segment extending from one side of a circle to the opposite, through the center point

Difference—the result of subtraction

Domain—the set of all x -values for a function

Equilateral Triangle—a triangle with all three sides equal and all three angles being 60°

Evaluate—to determine the value of an expression

Exponent—used to indicate the operation of repeated multiplication

Factor—an integer that divides into another integer equally

Function—ordered pairs (x, y) with exactly one y -value for any x -value

Hypotenuse—the side opposite the right angle of a right triangle

Improper Fraction—a fraction where the numerator is larger than the denominator

Integers—the set of numbers divisible by one without producing a remainder

Irrational Numbers—any number that cannot be expressed as a fraction

Isosceles Triangle—a triangle with two equal sides and equal opposite angles

Least Common Denominator—the smallest natural number that can be divided evenly by all denominators in the equation

Legs—in a right triangle, the two sides that are not the hypotenuse

Mean—the sum of all items divided by the number of items

Median—in a set of numbers arranged in order, the middle value or the mean of the two middle values

Mixed Number—a term that has both a whole number part and a fractional part

Mode—the most commonly occurring value in a set of values

Monomial—an algebraic expression with only one term

Natural Numbers—the set of positive integers starting with 1

Numerator—the top term of a fraction

Obtuse Angle—an angle with a measure between 90° and 180°

Origin—the intersection between the x - and y -axes of a coordinate graph

Parallel Lines—lines that never intersect regardless of how far they are extended

Parallelogram—a four-sided closed figure with opposite sides that are parallel and of equal length

Percentage—a fraction with a denominator of 100

Perimeter—the total distance around the outside of a polygon

Perpendicular Lines—two lines that intersect at a 90° angle

Polygon—a multi-sided closed plane figure

Polynomial—an algebraic expression with two or more terms

Power—used to indicate repeated multiplication

Prime Numbers—numbers that are evenly divisible only by themselves and one

Product—the result of multiplication

Proper Fraction—a fraction where the denominator is larger than the numerator

Quadrants—the four sections of a coordinate graph

Quadrilateral—a four-sided closed plane figure

Quotient—the result of division

Radius—a line segment extending from the center of a circle to any point on the circle

Range (1)—the difference between the largest and smallest numbers in a set

Range (2)—the set of all y -values for a function

Rational Number—any number that can be expressed as a fraction

Real Numbers—the set of rational and irrational numbers

Rectangle—a four-sided plane closed figure with opposite sides equal and 90° angles

Right Triangle—a triangle with a 90° angle

Root—a number that when raised to a certain power will yield a second given number

Scientific Notation—a number written as the product of a real number between 1 and 10 and a power of 10

Set—a group of numbers, elements, objects, etc.

Square (1)—the result when a number is multiplied by itself

Square (2)—a four-sided plane closed figure with all four sides equal and 90° angles

Square Root—a number that when raised to the second power will yield a second given number

Sum—the result of addition

Supplementary Angles—two angles with a sum of 180°

Term—an expression, either numerical or literal

Triangle—a three-sided closed plane figure

Variable—a symbol that is used to stand for a number

Vertex—a point at which two rays or sides of a polygon meet to form an angle

Volume—the space inside a solid, measured in cubic units

Whole Numbers—the set of positive integers including zero

X-axis—the horizontal axis of a coordinate graph

Y-axis—the vertical axis of a coordinate graph

